

A Simple Model for Time-Varying Expected Returns on the S&P 500 Index

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Abstract

This paper presents a parsimonious, implementable model for the estimation of the short- and long-term expected rates of return on the S&P 500 stock market Index. The model estimates a parametric form for the Market Price of Risk, the Sharpe Ratio, of the S&P 500 Index. In addition to short- and long-term risk-free rates of interest, the model’s empirical estimation makes use of two forward-looking measures: The economy’s *growth rate* estimate; and the option market’s (priced) implied volatility on the S&P 500 Index. The model accounts for past rates of return by modeling and estimating the impact of an assumed *increasing relative risk aversion*, which gives rise to an increased willingness to invest in risky assets as the realized rate of return for the recent past is “high.”

Specifically, conditioning on three variables — the risk-free rates of interest r_{1t} and $r_{30,t}$, the implied volatility VIX_t on the Index, and the *realized* S&P 500 Index rate of return over the past five–six years $S\&P\ 500_t/S\&P\ 500_{t-5,t-6}$ — the model generates prospective expected rates of return μ_t of the form

$$\mu_t = \begin{cases} r_{1t} + \left(\lambda_0 + \lambda_1 \frac{S\&P\ 500_t}{S\&P\ 500_{t-5,t-6}} \right) VIX_t & \text{for a } one\text{-year horizon} \\ r_{1t} + \widehat{\beta}(r_{Lt} - r_{1t}) + a_0 + a_1 VIX_t + a_2 VIX_t \frac{S\&P\ 500_t}{S\&P\ 500_{t-5,t-6}} + a_3 \frac{S\&P\ 500_t}{S\&P\ 500_{t-5,t-6}} & \text{for the long-term} \end{cases}$$

where the current prevailing Sharpe Ratio is $\lambda_0 + \lambda_1 (S\&P\ 500_t/S\&P\ 500_{t-5,t-6})$, and λ_0 , λ_1 , $\widehat{\beta}$, a_0 , a_1 , a_2 and a_3 are coefficients estimated from the data.

1 Introduction

Since financial markets began modern trading of equity securities, financial economists have struggled to understand the equity risk premium, both domestically and across international markets. Summarizing as it does the rate of return on investable risky wealth in the economy, the equity risk premium has constituted the compensation for equity investments; it has been studied as measures of the economy’s well-being, and as a broad measure of the success of market-based economies.¹

Estimating the contemporaneous (conditional) expected rate of return on equity markets, and properly capturing its intertemporal variation, has been the source of more modern analyses. We seek to add to the literature by addressing a market-efficient estimate of time-varying expected returns, grounded in theory but parsimonious in application. We estimate a parametric form for the Market Price of Risk, or the Sharpe Ratio, of the S&P 500 Index. In addition to short- and long-term risk-free rates of interest, our empirical implementation makes use of two forward-looking measures: The economy’s growth rate estimate; and the option market’s (priced) implied volatility on the S&P 500 Index. Further, our model explicitly accounts for realized wealth levels by modeling and estimating an assumed increasing relative risk aversion, which gives rise to an increased willingness to invest in risky assets when the realized rate of return for the recent past is “high.”

Time-varying expected returns find at least two uses in practical applications. In the investments arena, various asset-allocation models require as input an expected rate of return on the “market portfolio.” In capital-budgeting decisions, a key ingredient is the asset’s expected rate of return, which uses as (one of the) inputs, the expected rate of return on the market.

The paper is now structured as follows. Section 2 reviews the literature on market risk premia and their intertemporal variation. Section 3 follows with a presentation of the theoretical models we wish to estimate. Section 4 contains the empirical tests, and their associated results. Section 5 includes robustness checks. Section 6 summarizes and concludes.

¹ For example, financial economists have sought to understand the U. S. equity risk premium in comparison to markets where a breakdown of trading occurred in the second and fourth decades of the 20th century.

2 The U. S. Equity Risk Premium

The current literature pertaining to the market risk premium starts with the CAPM model of Sharpe (1964) and Lintner (1965). In deriving a relationship between equity returns and a market-wide risk factor, these authors laid the foundation for the countless theoretical and empirical asset pricing articles that have become the stable of the financial economics literature. One of the most heavily tested and examined aspects of the model is the notion of the equity risk premium, or alternatively, the market price of risk or Sharpe ratio. Sharpe (1966) introduced this notion of reward to variability ratio in describing mutual funds, later denoted the Sharpe ratio or measure. As described in Sharpe (1994), and similar to our work here, the *ex-ante* Sharpe Ratio measures expected returns — in contrast to the distinct *ex-post* realized returns. This leads to several questions often debated within the literature: Estimation of the intertemporal equity price risk premium, and the factors that determine the magnitude of the premium. In this paper we attempt to address these issues.

A great deal of attention has been placed on examining the relationship between conditional volatility and the market risk premium. The theoretical implication of the CAPM is that there is positive relationship between the level of volatility and the size of the risk premium. However, the empirical evidence is mixed. Campbell (1987) and Glosten, Jagannathan and Runkle (1993) have documented a negative relationship between the conditional volatility and the risk premium, contrary to economic theory, while Harvey (1989), and Turner, Startz and Nelson (1989) found a positive relationship. Scruggs (1998) decomposed the CAPM model into a partial relation in a two-stage estimation, and was thus able to explain away the negative relationship of Campbell (1987) and Glosten et al. (1993). Brandt and Zhou (2004) attempt to resolve these differences in the literature regarding contemporaneous correlation by implementing a VAR technique. By incorporating time-varying volatility, their conclusions suggest that these differences can be explained by the conditional and unconditional correlations.

The evidence on time-varying expected returns has been demonstrated by the volatility ratio tests of LeRoy and Porter (1981), and the long-horizon autoregressions of Fama and French (1988a, b). These findings have been corroborated by documenting time-varying risk premium by Ferson and Harvey (1991) and Evans (1994). Campbell and Viceria (2005) take the time variation in expected returns a step further by suggesting that investors, particularly aggressive investors, may want to engage in market-timing (or tactical asset allocation) strategies aimed at maximizing short-term returns, based on the predictions of their return forecasting model. Still, there is considerable uncertainty about the degree of

asset return predictability, as noted by Pástor and Stambaugh (2001), making it hard to identify the optimal market timing strategy. Attempting to capture the time variation of expected returns has been extensively examined. Using a multi-beta asset pricing model, Ferson and Harvey (1991b) incorporate risk exposure to the market as well as the interest rate and inflation to explain realized returns. Others, such as Lewellen (1999, 2004), have used explanatory variables such as the dividend yield, short rate, term premium, Book-to-Market, and the default premium; however, in light of the statistical issues brought up in Boudoukh and Richardson (1993), Stambaugh (1999), and Ferson et al. (2003), the validity of the results are still in question.

Finally, in their highly influential piece, Mehra and Prescott (1985) first documented the equity premium *puzzle*. They found that the annualized rate of return on stocks in excess of the risk-free rate is higher than can be explained by the classical theories in financial economics by about 6.8%. Mehra (2003) further decomposed the fundamental pricing relationship and demonstrated that the growth rate of consumption does not vary enough to be consistent with the observed high equity premium. In calibrating the model, using upper-bound levels for risk-aversion generates a risk-free rate that is too high and a risk-premium that is too low. While this is troubling, Mehra (2003) points out that this is a quantitative puzzle, and that current theory is consistent. Many authors have attempted to resolve this puzzle, including Campbell and Cochrane (1999) and Constantinides (1990), by altering preferences, using incomplete markets, survivorship bias, and omitting rare events. As of yet, there is no current solution to this problem.

Our work will extend the prior knowledge by incorporating all three strands of this literature. By decomposing a simple model for valuing stocks, such as the dividend growth model, into various parts, and extracting the ex-ante market price of risk, we can explain the time-variation in the market price of risk with a simple measure of investor sentiment. This measure is negatively related to the premium, such that the higher the perceived wealth, the lower the market price of risk. However, and similar to the Mehra and Prescott (1985) findings, the predicted value of the expected return is still approximately 4%–5% less than the realized returns over the given evaluation period, consistent with the findings of Fama-French (2002). Using our measure, we are able to explain as much as 50% of the variation in expected returns; thus providing strong indication of potential peaks and troughs in the market.

3 The Models

Our model is a straightforward combination of the Gordon-Shapiro (1956) – Williams (1938) dividend-growth model and the Sharpe-Lintner security market line. The alternate models we postulate and test vary by their:

1. Interpretation of the Livingston/Philadelphia Fed short- and long-term growth rates
2. Use of the VIX implied vol value in terms of volatility
3. Examination of the long-term vs. one-year expected rates of return on the S&P 500

3.1 A *Short-Term Expected Return Model*

Consider first the interpretation of the 1-yr. Livingston growth rate $g_{1t} \equiv g_t$ as both a short-term dividend growth rate as well as the capital-gains component of the S&P 500. In that case, we have

$$\mu_t = \frac{D_{0t}(1 + g_{1t})}{P_t} + g_{1t}, \quad (1)$$

where

μ_t = Expected/required rate of return on the equity asset as of date t

P_t = Price of equity asset at date t

D_{0t} = Dividends payable over the past 12 mos., as of date t , and assumed to satisfy the relationship $D_{1t} = D_{0t}(1 + g_{1t})$

g_{1t} = One-year dividend growth rate as of date t *and* capital-gains forecast over the next twelve months

From the security market line, we have, at date t ,

$$\mu_t = r_{1t} + \lambda_t \sigma_t, \quad (2)$$

where

λ_t = Market Price of Risk, or Sharpe Ratio, as of date t

r_{1t} = One-year Treasury Bill rate of interest as of date t

σ_t = Equity asset's annualized volatility, as of date t

Assuming that $VIX_t = \sigma_t$, we can equate eq. (1) to eq. (2) to yield

$$\frac{D_{0t}}{P_t} (1 + g_{1t}) + g_{1t} = r_{1t} + \lambda_t VIX_t$$

which can be solved for λ_t :

$$\frac{(D_{0t}/P_t) (1 + g_{1t}) + g_{1t} - r_{1t}}{VIX_t} = \lambda_t \quad (3)$$

Assuming the observability of $\{D_{0t}, P_t, g_{1t}, r_{1t}, VIX_t\}$, the LHS of (3) is an observable datum.

The final step of this model is to provide a functional, estimable and empirically testable form for λ_t . To do so, we rely on financial theory for guidance. Specifically, we assume the representative investor exhibits increasing relative risk aversion, so that λ_t can be modeled as a function of investable (presumably, per capita) wealth. We proxy for that per capita wealth level by making λ_t a function of the past realized return on the S&P 500:

$$\lambda_t = \begin{cases} \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \\ \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \lambda_2 \left(\frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \right)^2 \end{cases} \quad (4)$$

where $\lambda_0 > 0$, $\lambda_1 < 0$ and $\lambda_2 > 0$ are coefficients to be estimated, for some value of T . The postulated negative sign of λ_1 reflects our assumption that, as $\text{S\&P } 500_t/\text{S\&P } 500_{t-T}$ increases and investors feel “wealthier,” their required compensation per unit standard deviation declines. When modeled in its *quadratic* form, the postulated $\lambda_2 > 0$ reflects a standard extension permitting decreasing marginal effect of wealth.

Thus, the empirical relationships we will test combine the two equations (3) and (4) to produce:

$$\frac{(D_{0t}/P_t) (1 + g_{1t}) + g_{1t} - r_{1t}}{VIX_t} = \begin{cases} \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \\ \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \lambda_2 \left(\frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \right)^2 \end{cases} \quad (5)$$

Interpreting this variant of the model as a test of a short-term, one-year Sharpe ratio — in contrast to the models in sections 3.2 through 3.4 below — we will seek to explore how well the simple model of the form (5) explains the time-varying changes in our proxy for expected returns, whether the estimated signs and magnitudes of $\{\lambda_0, \lambda_1, \lambda_2\}$ conform to

economic intuition. Given the results obtained in (5), the quadratic model's implied *ex-ante* expected rate of return on the S&P is given by

$$\mu_t = r_{1t} + \left[\lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \lambda_2 \left(\frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \right)^2 \right] \text{VIX}_t, \quad (6)$$

with the linear model obtaining with its respective $\{\lambda_0, \lambda_1\}$ parameters.

For completeness, we will contrast those results with analogous results for the intertemporal variation in *realized* returns.²

3.2 Two-Growth Rate Model

In taking explicit notice that post-June 1990 the Livingston data provide both a one-year growth GDP growth-rate forecast g_1 *as well as* a ten-year forecast g_{10} , the next model takes explicit cognizance of these two growth rates. Specifically, we now interpret these two growth rates as a one-year and infinite-maturity dividend growth rates, giving rise to the valuation model

$$P_t = \frac{D_{1t}}{\mu_t - g_{10,t}} = \frac{D_{0t}(1 + g_{1t})}{\mu_t - g_{10,t}},$$

which can be sequentially inverted to solve for $\lambda_t \equiv (\mu_t - r_{1t}) / \text{VIX}_t$:

$$\mu_t = \frac{D_{0t}(1 + g_{1t})}{P_t} + g_{10,t} \quad (7)$$

$$\lambda_t = \frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} \quad (8)$$

For the post-June 1990 for which the full set of data (i.e., $g_{10,t}$) are available, the linear and quadratic testable versions of (8) are respectively given by:

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} = \begin{cases} \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \\ \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \lambda_2 \left(\frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \right)^2 \end{cases} \quad (9)$$

The linear and quadratic *ex-ante* expected returns is once again given by expression (6), with the respective $\{\lambda_0, \lambda_1, \lambda_2\}$ parameters having been estimated using (9).

²A robustness test will be offered to examine measurement errors in the growth-rate g and a relaxation of whether VIX is the proper proxy for volatility.

3.3 Two-Growth Rate, “Term Structure-Adjusted” Model

In beginning to make the transition to a long-term expected return model, consider the issue of the *slope* of the term structure of interest rates, of which investors are presumably aware and one which they may well take into account in establishing equity required expected rates of return. Thus, consider a re-formulation of (9) which permits us to elicit from the data the combination of short- and long-term rates which investors contemplate.

Proceeding from (7), we have the risk premium equation

$$\mu_t - r_{1t} - \beta(r_{Lt} - r_{1t}) = (D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t} - \beta(r_{Lt} - r_{1t}), \quad (10)$$

where the long maturity can be $L \in \{10, 30\}$. Thus, by estimating which β best explains the market price of risk, we can infer how investors “optimally” choose their target interest rate (and maturity).³ We proceed to estimate β and infer its implication, in two steps:

1. If we divide the LHS of (10) by VIX_t , we obtain a term structure-adjusted measure of the market price of risk λ_t . Thus, we have

$$\lambda_t \equiv \frac{\mu_t - r_{1t} - \beta(r_{Lt} - r_{1t})}{VIX_t} = \frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t} - \beta(r_{Lt} - r_{1t})}{VIX_t}. \quad (11)$$

Having in (4) modeled the market price of risk as linearly and quadratically dependent on the wealth accumulation factor $S\&P\ 500_t/S\&P\ 500_{t-T}$, we can apply that model to the RHS of (11):

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t} - \beta(r_{Lt} - r_{1t})}{VIX_t} = \lambda_0 + \lambda_1 \frac{S\&P\ 500_t}{S\&P\ 500_{t-T}} \quad (12)$$

2. To obtain an empirical estimate of β in the linear model, we first transpose it to the RHS of (12):

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{VIX_t} = \lambda_0 + \lambda_1 \frac{S\&P\ 500_t}{S\&P\ 500_{t-T}} + \beta \frac{r_{Lt} - r_{1t}}{VIX_t}. \quad (13)$$

Substituting the estimated parameter $\hat{\beta}$ from (13) back into (12) produces the estimated $\{\lambda_0, \lambda_1\}$ parameters.

The estimated coefficients obtained from constraining $\beta = \hat{\beta}$ are of course identical to the ones obtained in the regression (13), but their *interpretation* is now different: We have

³Thus, if $L = 30$ and $\beta = 1$, then $r_{1t} + \beta(r_{Lt} - r_{1t}) = r_{Lt} = r_{30,t}$, in which case the “target maturity” is $L = 30$ years.

elicited the optimal term structure adjustment to the expected rate of return μ_t . Specifically, with this information in hand, the quadratic model's prospective date t expected rate of return is given by

$$\mu_t = r_{1t} + \beta(r_{Lt} - r_{1t}) + \left[\lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \lambda_2 \left(\frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \right)^2 \right] \text{VIX}_t.$$

3.4 A Long-Term Expected Rate of Return Model: Two-Growth Rate, Term Structure Adjustment, Stochastic Volatility Model

In seeking a long-term expected return model, we recognize the short-term, one-month nature of VIX. In accordance with the previous work of Doran and Ronn (2005) and others, for longer-term periods, especially those exceeding one year, investors are cognizant of the well-documented mean-reversion in VIX. This final model will take cognizance of this mean-reversion.

Incorporating stochastic volatility helps capture the long-run component of volatility, as well as the level to which volatility reverts. In doing so, the weight w on current versus long-run volatility can be determined endogenously. Accounting for mean reversion requires the adjustment of the volatility variable to

$$w \text{VIX}_t + (1 - w) \sqrt{\theta}, \quad (14)$$

instead of dividing through by VIX_t . As shown in Doran and Ronn (2005), the stochastic model for volatility changes is given by

$$d\sigma_t^2 = \kappa (\theta - \sigma_t^2) dt + \xi \sigma_t dz, \quad (15)$$

where κ is the speed of mean reversion, θ is the level to which volatility reverts, and ξ is the variation in volatility. The relationship between κ and w in (14) is given by the weighting $w = \exp\{-\kappa T\}$ for whatever T value investors have "in mind." Whereas this model has been empirically verified, for our purpose here we need not discretize (15), but rather use its analytical *implication* (14).

With the expression (14) replacing VIX_t in (13), the expression (13) becomes

$$\begin{aligned} \lambda_t &\equiv \frac{\mu_t - r_{1t} - \beta(r_{Lt} - r_{1t})}{w \text{VIX}_t + (1 - w) \sqrt{\theta}} = \frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t} - \beta(r_{Lt} - r_{1t})}{w \text{VIX}_t + (1 - w) \sqrt{\theta}} \\ &= \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \beta \frac{r_{Lt} - r_{1t}}{w \text{VIX}_t + (1 - w) \sqrt{\theta}} \end{aligned} \quad (16)$$

Since the parameters $\{w, \theta\}$ are unknown, the estimation procedure for (16) is altered. Multiplying through by the "blended" volatility measure $w \text{VIX}_t + (1 - w) \sqrt{\theta}$, we have

$$\begin{aligned}
\frac{D_{0t}(1+g_{1t})}{P_t} + g_{10,t} - r_{1t} &= \left(\lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \right) \left[w \text{VIX}_t + (1-w) \sqrt{\theta} \right] \\
&\quad + \beta (r_{Lt} - r_{1t}) \\
= \lambda_0 (1-w) \sqrt{\theta} + \lambda_0 w \text{VIX}_t &+ \lambda_1 w \text{VIX}_t \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \lambda_1 (1-w) \sqrt{\theta} \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \\
&\quad + \beta (r_{Lt} - r_{1t}) \\
\equiv a_0 + a_1 \text{VIX}_t &+ a_2 \text{VIX}_t \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + a_3 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \beta (r_{Lt} - r_{1t})
\end{aligned} \tag{17}$$

The regression formulation in (17) merits several comments:

1. The dependent variable is no longer λ_t , but rather the term-structured adjusted risk premium
2. The regression is not unconstrained, as there is a linkage amongst the regression parameters $\{a_0, a_1, a_2, a_3\} : a_1/a_2 = \lambda_0/\lambda_1 = a_0/a_3$
3. The regression formulation does not permit the distinct, separate identification of all variables of interest $\{\lambda_0, w, \theta, \lambda_1, \beta\}$ — the regression is in that sense underidentified — but it does permit their estimation in the form they are required in order to calculate the expected return $\mu_t : \{a_0, a_1, a_2, a_3, \beta\}$. To see this, note that the (linear model's) expected return μ_t is now given by:

$$\begin{aligned}
\mu_t &= r_{1t} + \hat{\beta} (r_{Lt} - r_{1t}) + \lambda_0 (1-w) \sqrt{\theta} + \lambda_0 w \text{VIX}_t \\
&\quad + \lambda_1 w \text{VIX}_t \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + \lambda_1 (1-w) \sqrt{\theta} \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} \\
\equiv r_{1t} + \hat{\beta} (r_{Lt} - r_{1t}) &+ a_0 + a_1 \text{VIX}_t \\
&\quad + a_2 \text{VIX}_t \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}} + a_3 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}}
\end{aligned}$$

4. The R^2 of the regression formulation (17) is not meaningful, in that it includes the regressor $r_{Lt} - r_{1t}$ on its *RHS*. Rather, using $\hat{\beta}$ as determined from regression (17), what *is* meaningful is the R^2 of $(D_{0t}/P_t)(1+g_{1t}) + g_{10,t} - r_{1t} - \hat{\beta}(r_{Lt} - r_{1t})$ regressed on the remaining RHS variables $\{\text{VIX}_t, \text{VIX}_t \cdot \text{S\&P } 500_t/\text{S\&P } 500_{t-T}, \text{S\&P } 500_t/\text{S\&P } 500_{t-T}\}$

3.5 Realized Returns

In order to properly compare and contrast the results obtained under (5), we will perform analogous results for realized returns, which will replace expected returns in the LHS of (5):

For monthly annualized *realized* returns given by $R_t^m \equiv \left[(\text{S\&P}_{t+1/12} + D_{t+1/12}) / \text{S\&P}_t \right]^{12} - 1$, we will perform tests of the type

$$\frac{R_t - r_t}{\text{VIX}_t} = \lambda_0 + \lambda_1 \frac{\text{S\&P } 500_t}{\text{S\&P } 500_{t-T}}.$$

This process will then be repeated for the three models outlined in eqs. (9),(13), and (17).

4 Empirical Results

4.1 Data

To derive the expected Sharpe ratio, or λ , daily prices of the S&P 500 and the VIX/VXO index were collected from CRSP and the CBOE respectively from January 1986 through December 2004. VIX is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices. Since its introduction, VIX has been considered the premier barometer of investor sentiment and prospective market volatility. The VXO index was substituted up until 1990 since there were no observations of VIX prior to that date. This was done in part to infer estimates of expected λ prior to the October 1987 crash.

The data for the dividend yield come from Standard and Poors dividend bulletin, which reports quarterly dividends. To construct the dividend yield/payout, the sum of the prior four dividends is calculated prior to dividing by the current level of S&P 500.⁴ For the short-term risk-free rate, daily 1-year Treasury bill yields were collect from the Federal Reserve. For the long-term rate, the 10-year T-note yields were used. In addition, a mixture of the 30-year and 20-year T-bill rate were collected to provide an alternative measure.⁵

To proxy for the long-term and short-term dividend growth rates, the Livingston Survey was used, which provides semi-annual GDP forecast from economists from industry, government, banking, and academia. The feasibility of using the Livingston data as an accurate forecasting tool has been examined by many authors, incorporated in over 98 studies as of 1997, noted in the summary piece by Croushore (1997).⁶ In deriving the implied short-term

⁴We had sought to use futures contracts on the S&P to obtain implied future dividend yields. Unfortunately, active futures contracts do not extend the full one-year maturity required to calculate such an implied dividend yield. Using long-dated LEAPS to infer the value of long-dated S&P futures contracts might provide an implied *ex-ante* dividend yield, albeit in this case one driven by the risk-neutral rather than statistical expectations.

⁵The last observed date for the 30-year T-bill is on 2/15/2002. All subsequent dates use the 20-year T-bill yields.

⁶Most of the noted flaws in using the Livingston data have focused on the CPI forecasts,

dividend/capital gains growth rate, the forecast for the current year and next year of nominal GDP are used to construct a one year expected growth rate.⁷ Since the data’s frequency is semi-annual, this potentially gives rise to estimation problems if the growth forecast is not constant within the semi-annual period. This potential measurement error will be accounted for in robustness checks presented in section 5 and presented in detail in the Appendix. For the long-term growth rate, the Livingston data provides a 10-year forecast. However, one particular drawback in using the 10-year forecast is that the data only spans June 1990 through December 2004, reducing the number of observations and eliminates the October 1987 crash period. The summary statistics for all data are provided in Table 1.

4.2 Estimation Procedure for Short-term Expected Returns

To explain the time-variation in the expected Sharpe ratio, a measure of perceived wealth was constructed. Our proxy for investor sentiment is the ratio of the current level of the S&P 500 to some prior level of the index. Our hypothesis contends that there is a negative relationship between this ratio and the market price of risk. If the intercept term is positive, as should be expected, then the higher this ratio is, the lower the expected rate of return required to satisfy aggregate investor preferences.

Recalling the hypotheses stated in eq. (5)

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{1t} - r_{1t}}{\text{VIX}_t} = \begin{cases} \lambda_0 + \lambda_1 x_t + e_t \\ \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 + e_t \end{cases}$$

where

x_t is alternately defined to be $\text{S\&P}_t/\text{S\&P}_{t-5}$, $\text{S\&P}_t/\text{S\&P}_{t-6}$ or $\text{S\&P}_t/\text{S\&P}_{t-5,t-6}$

$\text{S\&P}_{t-5,t-6}$ is the *average value* of the S&P index over the time period between five and six years ago

This formulation provides an intuitive representation of investors’ perceptions. It is possible that investors have shorter, or perhaps longer, time horizons, but going back five to six years captures a limited-memory aspect in that the market is aware of past market highs and lows, but its memory is finite.⁸

and not GDP. In particular, Dokko and Edelstein (1989) find that the Livingston stock market surveys are unbiased estimators of realized stock returns.

⁷Base values are not used as there are discrepancies in the data as noted by the Philadelphia Fed.

⁸Such limited-memory allows us to indirectly model the *per capita* wealth which we seek to proxy, and which a long-term upward-drift in the S&P would fail to capture.

The mean value over the period for $S\&P_t/S\&P_{t-5}$ is 1.79, equivalent to an annual return of 12.4%. The maximum and minimum values were 3.21 (26.2%) and .76 (−5.31%), respectively. In testing the hypothesis (5), a quadratic formulation was included to capture decreasing marginal wealth effects.

The results reported in Table 2 confirm the positive intercept and negative slope coefficients consistent with our hypothesis. Both slope and intercept terms are statistically significant, with a high R^2 . It appears that six-year time horizon has the best overall performance, with an R^2 exceeding .51. Figure 1 demonstrates the relationship through time between the predicted values, $\tilde{\lambda}$, using the regression coefficient estimates, and the expected values, $E(\lambda_t)$, from eq. (3). As can be seen the model has tremendous explanatory power in capturing the short-term variation in expected returns.

To interpret the results, using $S\&P_t/S\&P_{t-5}$ the coefficient estimates from eq. (5) suggest a mean expected λ of .162. Using $S\&P_t/S\&P_{t-6}$ and $S\&P_t/S\&P_{t-5,t-6}$, the expected market price of risk are .164 and .163, respectively. The quadratic results fusing the three measures produce a mean λ of .155, .155, and .156. Interestingly, regardless of the dependent variable chosen, the expected Sharpe ratio is around .16 even though there are significant differences in model performance. Using these expected values, and multiplying by VIX, we can derive estimates for the time-varying expected risk premia. As shown in Figure 2, the expected risk-premia has varied from a high of 14.3% on 10/21/87 (2 days after the crash) to a low of −1.8% on 4/11/00 (12 trading days after the S&P 500 high watermark of 1527.46). On average the estimated expected risk premia over this period is 3.19%. Combined with the short-term risk-free rate corresponds to a expected rate of return of 8.34%, which is 4.3% less than the realized return on the S&P 500 calculated over the same period.⁹

Examining the maximum and minimum values for $S\&P_t/S\&P_{t-5}$, results in Sharpe ratios of −.043 and .313. The −.043 negative value is an indication that at high perceived wealth levels, investors are willing to accept negative equity risk premium, an apparent manifestation of risk-seeking behavior. Such a phenomenon is not accommodated in our standard utility functions: Standard utility functions permit investors to devote an increasing proportion of their wealth to the risky asset as their wealth increases, but they do not give rise to risk-seeking behavior. Of course, it may well be that, in practice, a sufficiently long run of positive returns on the S&P does indeed give rise to seemingly risk-seeking behavior: Such a negative equity risk premium may, in other words, constitute a sufficient condition for a “bubble.”

⁹The realized returns were calculated using annualized daily return and dividends.

4.3 Estimation of Two-Growth Rate Model

The previous estimation results were predicated on using only short-term interest and growth rates to arrive at an expected market price of risk. It more likely that multiple rates are used in deriving current prices, and is demonstrated in eq. (9). As outlined in section 3.2, incorporating both a long- and short-term growth rate will change the calculation of the expected market price of risk, but will not change the econometric specification. Thus, the regression outlined in eq. (5) can be used to test the multiple growth rate specification.

Since there appears to be little difference in the results in using multiple definitions of x_t , all remaining estimation will use $S\&P_t/S\&P_{t-5,t-6}$ as the proxy for per capita wealth level. The results of the regressions on the new dependent variable are shown in Table 3. Incorporating the long-term growth rate into the model provides about a 4% increase in performance, with an inferred mean value for the expected market price of risk equal to .199. This value is higher than the estimate from model 3.1, but not surprising given that long-term growth rate is higher on average than the short-term rate. However, the small relative performance improvement suggests that using multiple growth rates have little explanatory power beyond using only one growth rate. This is demonstrated in Figure 2, where the expected risk premium inferred from model 3.1 and model 3.2 exhibit almost identical time variation.

4.4 Estimation of Two-Growth Rate, “Term Structure-Adjusted” Model

By allowing for multiple growth rates, but using only a short-term risk-free rate, we were able to isolate the impact of multiple growth rates on model performance. While the results are interesting, the model is misspecified since a long-term growth rate should be accompanied with a long-term risk-free rate. As shown in section 3.3, the resulting model incorporates both the long- and short-term risk-free rate, but requires additional estimation.

The following regression will capture the weight placed on both the long ($r_{L,t}$)- and short-term ($r_{S,t}$) risk-free rates

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{VIX_t} = \begin{cases} \lambda_0 + \lambda_1 x_t + \beta \frac{r_{L,t} - r_{S,t}}{VIX_t} + e_t \\ \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 + \beta \frac{r_{L,t} - r_{S,t}}{VIX_t} + e_t \end{cases}$$

Estimating β will reveal the sensitivity to the term-premium investors incorporate within their expectations. To estimate β , the following estimations will use two proxies for the

long- and short-term rates. First, the 10-year rate and 1-year Treasury rates will be used as the initial long- and short-term rate respectively. Second, the long-term rate will encompass the 30-year rate up through 2002, and then 20-year rate after that, to capture additional term-premium that is not contained in the 10-year note.

The estimation procedure requires two steps. First, the model's first stage, outlined in eq. (13), is estimated to capture the coefficient estimates for β . After that coefficient is estimated, the second stage is estimated, where the dependent variable is adjusted by $\widehat{\beta}(r_{L,t} - r_{S,t})/VIX_t$ as shown in eq. (12). The results of each estimation using both proxies for the long- and short-term rates can be found in Table 4.

What is immediately obvious is the performance of the model when a term-structure adjustment is incorporated. It is interesting to note that regardless of the proxy chosen for the long-term rate, the coefficient on the term-structure spread is greater than one. This suggests investors are extremely sensitive to the spread between long and short rates, with higher spreads inducing greater required rates of return for equivalent levels of risk.

The second-stage results test the original model, but have adjusted the dependent variable by accounting for the term-premium. While the model performance is moderate, the coefficients on the wealth premium have maintained some explanatory power, sign direction, and significance. The resulting expected market price of risk after controlling for the yield spread suggests a mean value of .059, significantly less than the findings for the short-term model. However, as shown in Figure 2, the expected risk-premium demonstrates substantially less variation once the term-premium has been accounted for. This finding is entirely intuitive: The long-term risk premium should indeed be relatively *insensitive* to short-term fluctuations.

4.5 Incorporating Stochastic Volatility in the Estimation of a Long-Term Expected Risk Premium

The final estimation accounts for the mean-reversion in volatilities. As such, all prior estimations have used the current level of VIX for volatility. Estimating the model in section 3.4 can be done in two ways. It is possible to first estimate the mean reverting parameters as given in Doran and Ronn (2005), and then use the resulting mean-reverting parameters to find the market price of risk. However, given our model, it is possible to estimate both parameters without a separate estimation. Given eq. (17) we have the first-stage regression of the form of

$$\frac{D_{t,0}(1 + g_{S,t})}{P_t} + g_{L,t} - r_{1t} = a_0 + a_1 VIX_t + a_2 VIX_t x_t + a_3 x_t + \widehat{\beta}(r_{L,t} - r_{S,t}) + e_t$$

where the coefficients are given by

$$\begin{aligned} a_0 &= \lambda_0(1-w)\sqrt{\theta} \\ a_1 &= \lambda_0 w \\ a_2 &= \lambda_1 w \\ a_3 &= \lambda_1(1-w)\sqrt{\theta} \end{aligned}$$

Similar to the previous section, after solving for the coefficient on the term-premium, the dependent variable is adjusted by $\hat{\beta}(r_{L,t} - r_{S,t})$, and the regression is re-run. In addition, there is one non-linear constraint,

$$\frac{a_1}{a_2} = \frac{a_0}{a_3},$$

that must be imposed on both regressions. The results for both regressions using the two proxies for the long- and short-term risk-free rates are shown in Table 5.

The parameter values are not directly comparable to the other models, but suggest a fixed component to the long-run risk-premium of around 2.7%. The mean value for the risk-premium is 1.2% given the mean value of x_t and VIX of 1.89 and 21% respectively. As shown in Figure 3, the model suggests that even at the highest wealth levels, the risk-premium never falls below zero, reaching a minimum value on 3/28/2000 of 0.03%. This is in sharp contrast to the short-term findings, which found negative short-term expected Sharpe ratios. These difference highlight why there are bubbles and troughs over short-term intervals, while over the long-term, the market produces positive expected risk premiums.

In terms of performance, the findings for the first-stage regression are similar as those for Model 3.3. However, there is vast improvement in the second-stage regression suggesting the importance of accounting for mean-reversion in implied volatility. While it is not possible to infer the true value of w , a back-of-the-envelope calculation assuming a mean-value for long-run volatility of 21% and $r_L = r_{30}$, suggests a value of less than 10% on current volatility.¹⁰ This is quite surprising, as it suggests that investors have a long-term volatility perspective and are relatively insensitive to current values of volatility.¹¹ In this light, our ability to explain those relatively-minor changes in the long-term risk premium is of particular interest.

This is highlighted in Figure 2. By controlling for the mean-reverting nature of VIX, we have reduced the variation in the expected risk-premia beyond model 3.3, demonstrating an almost permanent component, which only fluctuates moderately with perceived levels of

¹⁰The inferred weight is calculated as $w = \frac{\sqrt{\theta}}{a_0/a_1 + \sqrt{\theta}}$.

¹¹This may, in part, explain why it has been difficult to attain a consensus on the correlation between volatility and the market risk premium.

wealth and contemporaneous values of VIX. Figure 4 decomposes the total rate of return implied by model 3.4 into two components, the blended risk-free rate and the expected risk-premium. As can be seen, the expected risk premium is a small component of the rate of return, and demonstrates relatively little time-variation. By comparison, the blended risk-free rate, which includes the short-term rate plus term-premium as estimated by the unconstrained regression, captures most of the time-variation. As several authors have noted, there was a general concern that the risk-premium was declining over recent years. However, the evidence here seems to suggest the contrary. Since the tech-bubble burst in 2000, the expected risk premium is on the rise; what has fallen is the spread between long and short term treasuries.

4.6 Realized Returns

We now wished to test all four models on realized returns. Given the current state of the literature on the equity premium puzzle, our expectation was that each model should perform quite poorly, which would be consistent with other empirical findings and the notion of unpredictability of asset returns. In addition, we wanted to demonstrate the realized Sharpe ratio was higher than the expected, confirming the findings of Mehra and Prescott (1985). The realized returns for day t were calculated in two ways:

$$R_t^m \equiv \left(\frac{S\&P_{t+1/12} + D_{t+1/12}}{S\&P_t} \right)^{12} - 1$$

$$R_t^a \equiv \left(\frac{S\&P_{t+1} + D_{t+1}}{S\&P_t} \right) - 1$$

where R_t^m is a monthly return and R_t^a is an annual return. $S\&P_{t+1/12}$ is the level of the S&P 500 one month ahead and $D_{t+1/12}$ is dividend payout divided by 12. We have calculated one month returns since VIX is a one-month estimate of implied volatility. One year returns are also calculated since typical holding periods are longer than one-month, and one month variation may incorporate short-term shocks outside of our current model. The annualized value of R_t^m in the one-month case is then adjusted by the annualized risk-free rate and VIX to create a realized market price of risk:

$$\lambda_t = \frac{R_t - r_t}{VIX_t}$$

This value is then regressed on the same independent variables as in the prior section to make the interpretation across realized versus expected market prices of risk comparable.

The results and estimation methodology reported in Tables (6-7) are analogous to those reported in sections 4.2-4.5, but using realized returns as the dependent variable.

Regardless of holding period, in each model, the R^2 is lower, except for the constrained regression of model 3.3 using 1-year holding period returns. This is interesting as it suggests there is limited explanatory power in the term-premium as compared to the expected results. While the variability in the expected Sharpe ratio is a function of the implied volatility, growth rate, risk-free rate, and dividend yield, the variability in realized Sharpe ratio appears unrelated to these factors. More appropriately, these results seem to suggest that there are additional factors that have yet been identified or are a result of random error.

The coefficient estimates from the regression imply a realized Sharpe ratio of .41 for each of the three models and is consistent with the simple historical calculation. The finding for the realized Sharpe ratio is almost 2.5 times greater than the expected Sharpe ratio over the same period. This results in a difference in the risk premia of roughly 4%. This is less than the Mehra and Prescott (1985) finding, where their reported difference between the realized and expected premium was roughly 6%.¹² The differences in expected, model predicted (using Model 3.1), and realized returns are shown in Table 8.

Annualizing the monthly returns by a factor of 252/22, we observe an annualized difference between expected versus realized return of about 4%. The regression's predicted difference is 4.4%. What is most revealing is how the results here again highlight the difference in predicting expected versus realized returns. There is little to no difference in the mean, standard deviation, minimum, and maximum values in the predicted and actual expected returns, while there are drastic differences in the predicted and actual realized returns.¹³ This suggests that the model effectively captures the ex-ante performance in the market, but reveals no information on ex-post returns. Needless to say, this result seems consistent with market efficiency, as there is a little to no relationship between ex-ante forecasting, and ex-post results. The question remains to whether this inability to capture realized returns is a function of model imperfection or random error. If it is the latter, this reaffirms the notion of unpredictability of asset returns.

¹²This may be a direct result of the 2000–2004 period, where the average difference is actually negative and is not included in Mehra (2003).

¹³The rolling 1-year realized returns report a similar mean, but an annualized standard deviation of 16.2%, a minimum value of -32.6% , and a maximum of 52.6% .

5 Modeling the Measurement Error in g_t and VIX

In using the data in this fashion, it is possible to introduce heteroscedasticity since there is the potential measurement error in the infrequently-observed growth rate. To address this problem, we adjust for the measurement error as shown in Appendix A and re-run equations (5) on the prior dependent variables. The resulting specification eliminates the intercept term (by dividing through by $\hat{\sigma}_{t,\epsilon}$, and results in a homoscedastic regression. The results of this regression for both expected and realized Sharpe ratios using all three measures of investor sentiment can be found in Table 9.

The findings are similar to the findings in the prior tables. The inferred Sharpe ratios for the expected and realized returns are around .14 and .41, respectively. In addition, the coefficients estimated using the heteroscedasticity adjustment are essentially the same as those in the standard OLS regression. We conclude that measurement error in the variables is of little concern.

6 Conclusion

This paper presented a parsimonious, easily-implementable model for the estimation of the short- and long-term expected rates of return on the S&P 500 stock market Index. Using as our primary variable of interest the Market Price of Risk, or Sharpe Ratio, of the S&P 500 Index, we used as predictive variables the risk-free rate of interest, the economy's growth rate estimate, and the option market's implied volatility on the S&P 500 Index. The model explicitly accounted for an assumed increasing relative risk aversion by incorporating and estimating the impact of past S&P 500 returns.

Conditioning on four variables — the risk-free rate of interest r_t , the slope of the yield curve $r_{30,t} - r_{1,t}$, the implied volatility VIX_t on the Index, and the *realized* S&P 500 Index rate of return over the past five–six years $S\&P\ 500_t/S\&P\ 500_{t-5,t-6}$ — the model generated expected rates of return μ_t given by expressions of the form

$$\mu_t = \begin{cases} r_{1t} + \left(0.46 - 0.162 \frac{S\&P\ 500_t}{S\&P\ 500_{t-5,t-6}}\right) VIX_t & \text{for a } one\text{-year horizon} \\ r_{1t} + 1.158 (r_{30,t} - r_{1t}) + 0.0257 + 0.0094 VIX_t \\ \quad - 0.00282 VIX_t \frac{S\&P\ 500_t}{S\&P\ 500_{t-5,t-6}} - .00772 \frac{S\&P\ 500_t}{S\&P\ 500_{t-5,t-6}} & \text{for the long-term} \end{cases} \quad (18)$$

In examining the implications of (18) for short- and long-term expected rates of return, we find that:

1. Short-term expected rates of return are quite volatile, due to changes in VIX, the term structure of interest rates $\{r_{1t}, r_{30,t}\}$ and the accumulated wealth factor $S\&P\ 500_t/S\&P\ 500_{t-5,t-6}$
2. The *behavior* of the short-term μ_t presents an interesting “history” of the past twenty years, with the risk premium peaking immediately subsequent to the 1987 stock market crash and reaching a low point — a *negative* risk premium just as the stock market reached its recent March 2000 high-water, possibly “bubble,” mark
3. The long-term expected risk premium is remarkably stable, as indeed befits a long-term predictor of the excess return on the U. S. stock market: Any transitory effects would be *expected* to dissipate in the long-term.
4. Whereas the long-term risk premium unsurprisingly reached a low point in the March – April 2000 time period, it remained slightly positive and never fell into negative territory

With respect to the relationship between expected and realized returns, we find the work Fama and French (2002), “The Equity Premium,” particularly relevant to our results. Quoting from their Abstract:

“We estimate the equity premium using dividend and earnings growth rates to measure the expected rate of capital gain. Our estimates for 1951 to 2000, 2.55% and 4.32%, are much lower than the equity premium produced by the average stock return, 7.43%. Our evidence suggests that high average return for 1951 to 2000 is due to a decline in discount rates that produces a large unexpected capital gain. Our main conclusion is that the average stock return of the last half-century is a lot higher than expected.”

Our work has used *prospective* data on growth rates and volatility for the time period available, Jan. 1986 to Dec. 2004, with a parsimonious expected-return model, and come to starkly similar results.

In conclusion, this suggests to us two qualitative results:

1. A negative implied equity risk premium, such as manifested themselves (in our data period) in Oct. 1987 and the 2000 period are strongly suggestive of “irrational exuberance” giving rise to unsustainably high asset prices.

2. Overall, in this period Jan. 1986 to Dec. 2004 there were positive shocks to the system that resulted in realized returns exceeding their expected values. While it is tempting to suggest two of these shocks were the “peace dividend” following 1991 and the productivity shocks induced by improved computer technology in the '90's, such attribution must at this time remain speculative.

A Modeling the Measurement Error in g_t and VIX

1. Recall the basic equation we are examining is:

$$\frac{D_{0t}(1+g_t)}{P_t} + g_t = r_t + \lambda_t \sigma_t. \quad (19)$$

x_t is our wealth-relative variable, e.g., $x_t = \text{S\&P}_t / \text{S\&P}_{t-6}$ or $x_t = \text{S\&P}_t / \text{S\&P}_{t-5}$. Now, combine (19) with

$$\lambda_t = \lambda_0 + \lambda_1 x_t$$

If we rearrange (19), we obtain

$$\frac{1}{\sigma_t} \left(\frac{D_{0t}}{P_t} + 1 \right) g_t + \frac{D_{0t}}{\sigma_t P_t} - \frac{r_t}{\sigma_t} = \lambda_0 + \lambda_1 x_t \quad (20)$$

2. If realized vol at date t is σ_t , then for a negative market price of vol risk, we have $\text{VIX}_t > \sigma_t$. Assume then that

$$\sigma_t = a \text{VIX}_t e_V$$

for some $a < 1$ and an error term e_V satisfying $E(\ln e_V) = 0$

3. Because of the quarterly (hence less than monthly) observation frequency for the growth rate g_t , assume the growth rate is measured with error, resulting in the substitution in eq. (20) of g_t with $g_t + e_g$
4. Substituting these two expressions into (20) results in:

$$\frac{1}{a \text{VIX}_t e_V} \left(\frac{D_{0t}}{P_t} + 1 \right) (g_t + e_g) + \frac{D_{0t}}{a \text{VIX}_t e_V P_t} - \frac{r_t}{a \text{VIX}_t e_V} = \lambda_0 + \lambda_1 x_t + e_t,$$

where e_t is the regression's error term. Now, multiplying through by $a e_V$ results in:

$$\frac{1}{\text{VIX}_t} \left(\frac{D_{0t}}{P_t} + 1 \right) (g_t + e_g) + \frac{D_{0t}}{\text{VIX}_t P_t} - \frac{r_t}{\text{VIX}_t} = \lambda_0 a e_V + \lambda_1 a x_t e_V + a e_V e_t$$

Transpose the e_g term to the RHS:

$$\begin{aligned} \frac{1}{\text{VIX}_t} \left(\frac{D_{0t}}{P_t} + 1 \right) g_t + \frac{D_{0t}}{\text{VIX}_t P_t} - \frac{r_t}{\text{VIX}_t} &= \lambda_0 a e_V + \lambda_1 a x_t e_V + a e_V e_t \\ &\quad - \frac{1}{\text{VIX}_t} \left(\frac{D_{0t}}{P_t} + 1 \right) e_g. \end{aligned} \quad (21)$$

Clearly, regression (21) is subject to heteroscedasticity, since (with the relevant statistical assumptions)

$$\begin{aligned} \text{Var} [\lambda_0 a e_V + \lambda_1 a x_t e_V + a e_V e_t - \frac{1}{\text{VIX}_t} \left(\frac{D_{0t}}{P_t} + 1 \right) e_g] &= \\ &= \left(\lambda_0^2 a^2 + \lambda_1^2 a^2 x_t^2 \right) \sigma_V^2 + \Sigma_1^2 + \frac{1}{\text{VIX}_t^2} \left(\frac{D_{0t}}{P_t} + 1 \right)^2 \sigma_g^2 \\ &\equiv \Sigma^2 + \lambda_1^2 a^2 x_t^2 \sigma_V^2 + \frac{1}{\text{VIX}_t^2} \left(\frac{D_{0t}}{P_t} + 1 \right)^2 \sigma_g^2, \end{aligned}$$

which shows the heteroscedasticity induced by x_t and $y_t \equiv \frac{1}{\text{VIX}_t} \left(\frac{D_{0t}}{P_t} + 1 \right)$.

5. Adjusting for this heteroscedasticity is not simple, since it has a constant Σ as well as time- t dependent variables x_t and y_t :

The solution we implement, albeit one that is cumbersome, is the following:

- (a) Estimate σ_g from a time-series of g_t 's: $\hat{\sigma}_g = \text{Std. Dev.}(g_t)$
- (b) Estimate σ_V from a regression of $\ln(\sigma_t/\text{VIX}_t)$ on a constant, which simultaneously produce an estimate of \hat{a} as well as $\hat{\sigma}_V$
- (c) Obtain an estimate of regression (21) slope coefficient $\widehat{\lambda_1 a}$ by running an OLS version of the regression (21)
- (d) By (initially) setting $\Sigma \equiv 0$, obtain an estimate of the date t regression error (22) $\hat{\sigma}_t$ by substituting $\hat{\sigma}_g$, $\widehat{\lambda_1 a}$ and $\hat{\sigma}_V$
- (e) Deflate both LHS and RHS of (21) by this estimate of $\hat{\sigma}_t$, then run the regression (21)
- (f) Calculate the std. dev. of the error term in that regression ϵ_t , $\hat{\sigma}_\epsilon$. On average, we would expect that

$$\hat{\sigma}_\epsilon^2 = \Sigma^2 + \frac{1}{T} \left[\sum_t \lambda_1^2 a^2 x_t^2 \sigma_V^2 + \frac{1}{\text{VIX}_t^2} \left(\frac{D_{0t}}{P_t} + 1 \right)^2 \sigma_g^2 \right]. \quad (22)$$

- (g) In (22), if $\Sigma \geq 0$, substitute Σ into (21), deflate by the new $\hat{\sigma}_t$ and run regression (21) one more time

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Table 1: Summary Statistics

Table 1 reports the summary statistics of the variables from January 1986 through December 2004

Variable	Obs	Mean	Stdev	Min	Max
Dividend Yield	4795	2.4%	0.8%	1.1%	4.0%
VIX	4795	20.6%	7.6%	9.3%	150.2%
Treasury Yield 1-Year	4795	5.1%	2.1%	0.9%	9.9%
10-Year	4795	6.5%	1.5%	3.1%	10.2%
30-Year	4795	6.8%	1.5%	3.1%	10.3%
Growth Rate					
1-year	4795	5.6%	1.0%	2.7%	7.3%
10-year	3679	5.9%	1.0%	5.0%	6.7%
$\frac{S\&P_t}{S\&P_{t-5}}$	4795	1.79	0.55	0.76	3.21
$\frac{S\&P_t}{S\&P_{t-6}}$	4795	2.01	0.54	0.92	3.43
$\frac{S\&P_t}{S\&P_{t-5,t-6}}$	4795	1.89	0.54	0.86	3.30

Table 2: Estimation of Model 3.1: One-Growth Rate, Short-Term Interest Rate, VIX Model

Table 2 reports the parameter estimates of the OLS regression of eq (5) shown below. The market price of risk is inferred from the dividend-growth model specified in eq. (1) and is regressed on x_t , three proxies of “perceived wealth”: (1) $S\&P_t/S\&P_{t-5}$, (2) $S\&P_t/S\&P_{t-6}$, and (3) $S\&P_t/S\&P_{t-5,t-6}$. Each panel contains linear and quadratic regressions. t -stats are listed in parentheses.

$$\begin{aligned} \text{Linear Relationship: } & \frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{1t} - r_{1t}}{VIX_t} = \lambda_0 + \lambda_1 x_t + e_t \\ \text{Quadratic Relationship: } & \frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{1t} - r_{1t}}{VIX_t} = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 + e_t \end{aligned}$$

	Model 1: $x_t = \frac{S\&P_t}{S\&P_{t-5}}$		Model 2: $x_t = \frac{S\&P_t}{S\&P_{t-6}}$		Model 3: $x_t = \frac{S\&P_t}{S\&P_{t-5,t-6}}$	
x_t	-0.146 (69.49)**	-0.254 (19.98)**	-0.169 (89.38)**	-0.293 (24.77)**	-0.162 (84.36)**	-0.253 (22.28)**
x_t^2		0.029 (8.80)**		0.03 (11.13)**		0.023 (8.44)**
λ_0	0.424 (96.09)**	0.517 (45.40)**	0.504 (106.82)**	0.623 (50.62)**	0.469 (104.36)**	0.552 (50.11)**
Obs	4795	4795	4795	4795	4795	4795
R^2	40.2%	41.0%	51.4%	51.7%	47.1%	48.2%

Robust t -statistics in parentheses

* significant at 5%; ** significant at 1%

Table 3: Estimation of Model 3.2: Two-Growth Rate, Short-term Interest Rate, VIX Model

Table 3 reports the parameter estimates of the OLS estimation of Model 3.2 given in eq. (9). The dependent variable is the market price of risk computed using both one-year g_{1t} and ten-year $g_{10,t}$ growth rates. The estimation period is from June 1990 through December 2004. $x_t = \frac{\text{S\&P}_t}{\text{S\&P}_{t-5,t-6}}$. t -stats are listed in parentheses.

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + e_t$$

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 + e_t$$

	Linear Model	Quadratic Model
x_t	-0.162 (93.24)**	-0.111 (9.08)**
x_t^2		-0.013 (4.32)**
λ_0	0.506 (117.81)**	0.46 (41.07)**
Obs	3679	3679
R^2	51.27%	51.43%

Robust t -statistics in parentheses

* significant at 5%; ** significant at 1%

Table 4: Estimation of Model 3.3: Two-Growth Rate, Blended Interest Rates, VIX Model

Table 4 reports the parameter estimates of the two-stage estimation of Model 3.3 given in eqs. (12-13). In the first-stage regression, the dependent variable is adjusted by $-\hat{\beta}(r_L - r_1)/\text{VIX}_t$. The dependent variable is the two-growth rate market price of risk, $[(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}]/\text{VIX}_t$. The estimation period is from June 1990 through December 2004. $x_t = \text{S\&P}_t/\text{S\&P}_{t-5,t-6}$. t -stats are listed in parentheses.

First Stage:

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + \beta \frac{r_{L,t} - r_{1t}}{\text{VIX}_t} + e_t$$

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 + \beta \frac{r_{L,t} - r_{1t}}{\text{VIX}_t} + e_t$$

Second Stage:

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} - \hat{\beta} \frac{r_{L,t} - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + e_t$$

$$\frac{(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}}{\text{VIX}_t} - \hat{\beta} \frac{r_{L,t} - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 + e_t$$

As explained in the text, the R^2 for the first-stage is not meaningful, but it is for the second-stage's bold-faced values.

	$r_L = r_{10}$ and $r_S = r_1$				$r_L = r_{30}$ and $r_S = r_1$			
	1st Stage	2nd Stage	1st Stage	2nd Stage	1st Stage	2nd Stage	1st Stage	2nd Stage
β	1.654 (96.33)**		1.659 (96.58)**		1.449 (124.76)**		1.448 (126.46)**	
x_t	-0.032 (20.19)**	-0.032 (36.24)**	-0.059 (8.83)**	-0.059 (9.01)**	-0.024 (18.11)**	-0.024 (31.51)**	-0.010 (1.76)	-0.010 (1.79)
x_t^2			0.007 (4.37)**	0.007 (4.28)**			-0.004 (2.62)**	-0.004 (2.58)**
λ_0	0.121 (27.68)**	0.119 (59.46)**	0.143 (19.39)**	0.143 (23.70)**	0.081 (22.16)**	0.081 (48.54)**	0.069 (10.76)**	0.069 (13.17)**
Obs	3679	3679	3679	3679	3679	3679	3679	3679
R^2	89.7%	16.1%	89.7%	16.5%	93.0%	13.9%	93.0%	14.1%

Robust t -statistics in parentheses

* significant at 5%; ** significant at 1%

Table 5: Estimation of Model 3.4: Two-Growth Rate, Blended Interest Rates, Blended Volatility Model

Table 5 reports the parameter estimates of the first- and second-stage linear estimation with non-linear constraints of Model 3.4 given in eq. (17). For both regressions the condition that $a_1/a_2 = a_0/a_3$ is imposed. Using the first-stage estimate $\hat{\beta}$, in the second-stage regression the dependent variable is adjusted by $-\hat{\beta}(r_L - r_S)$. The dependent variable is the risk premium $(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - r_{1t}$. The coefficients are equal to: $a_0 = \lambda_0(1 - w)\sigma$, $a_1 = \lambda_0w$, $a_2 = \lambda_1w$, $a_3 = \lambda_1(1 - w)\sigma$. The estimation period is from June 1990 through December 2004. t -stats are listed in parentheses.

First Stage

$$\frac{D_{0t}(1 + g_{1t})}{P_t} + g_{10,t} - r_{1t} = a_0 + a_1 \text{VIX}_t + a_2 \text{VIX}_t x_t + a_3 x_t + \beta(r_L - r_S) + e_t$$

Second Stage

$$\frac{D_{0t}(1 + g_{1t})}{P_t} + g_{10,t} - r_{1t} - \hat{\beta}(r_L - r_S) = a_0 + a_1 \text{VIX}_t + a_2 \text{VIX}_t x_t + a_3 x_t + e_t$$

	$r_L = r_{10}$ and $r_S = r_1$		$r_L = r_{30}$ and $r_S = r_1$	
	1st Stage	2nd Stage	1st Stage	2nd Stage
$\lambda_0(1 - w)\theta$	0.02896 (35.66)**	0.02891 (49.26)**	0.02571 (35.75)**	0.02573 (48.49)**
λ_0w	0.02899 (10.16)**	0.02899 (10.16)**	0.00939 (3.69)**	0.0094 (3.70)**
β	1.26908 (89.39)**		1.15754 (111.74)**	
λ_1w	-0.00814 (9.98)**	-0.00814 (9.99)**	-0.00282 (3.67)**	-0.00282 (3.69)**
$\lambda_1(1 - w)\theta$	-0.00813 (30.54)**	-0.00814 (42.01)**	-0.00771 (31.83)**	-0.00772 (41.49)**
Obs	3679	3679	3679	3679
R^2	90.62%	49.76%	93.23%	49.80%

Robust t -statistics in parentheses

* significant at 5%; ** significant at 1%

Table 6: Estimation of Realized Market Price of Risk using a 30-day Window

Table 6 reports the analogous parameter estimates using realized returns for all four models. The realized returns were calculated over a 30 day window, and the market price of risk was inferred using the implied volatility from VIX and the 1-year Treasury Bill. The realized market price of risk was then regressed on $x_t \equiv \text{S\&P}_t/\text{S\&P}_{t-5,t-6}$. All independent variables are calculated in the same fashion as those in the expected market price of risk regressions. For the calculation of β , $r_L = r_{10}$. t -stats are listed in parentheses.

$$\frac{\left[\left(\text{S\&P}_{t+1/12} + D_{t+1/12}\right)/\text{S\&P}_t\right]^{12} - 1 - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + e_t$$

$$\frac{\left[\left(\text{S\&P}_{t+1/12} + D_{t+1/12}\right)/\text{S\&P}_t\right]^{12} - 1 - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + \beta \frac{r_{Lt} - r_{1t}}{\text{VIX}_t} + e_t$$

$$\left(\frac{\text{S\&P}_{t+1/12} + D_{t+1/12}}{\text{S\&P}_t}\right)^{12} - 1 - r_{1t} = a_0 + a_1 \text{VIX}_t + a_2 \text{VIX}_t x_t + a_3 x_t + \beta (r_{Lt} - r_{1t}) + e_t$$

	Model 3.1 & 3.2		Model 3.3				Model 3.4	
			1st Stage	2nd Stage	1st Stage	2nd Stage	1st Stage	2nd Stage
x_t	-0.627 (9.24)**	0.66 (2.01)*	-0.95 (11.65)**	-0.95 (15.19)**	-0.433 (1.23)	-0.432 (1.23)		
x_t^2		-0.326 (3.90)**			-0.133 (1.49)	-0.133 (1.51)		
λ_0	1.609 (12.63)**	0.436 (1.4)	2.694 (13.30)**	2.693 (22.50)**	2.246 (6.55)**	2.244 (6.81)**		
$\lambda_0(1-w)\sigma$							0.33 (4.66)**	0.335 (5.67)**
$\lambda_0 w$							0.761 (2.63)**	0.76 (2.64)**
$\lambda_1 w$							-0.256 (2.60)**	-0.255 (2.61)**
$\lambda_1(1-w)\sigma$							-0.111 (4.51)**	-0.112 (5.36)**
β			-6.171 (9.33)**		-6.269 (9.34)**		-5.449 (6.71)**	
Obs	4775	4775	3657	3657	3657	3657	3657	3560
R^2	1.88%	2.14%	3.05%	5.47%	3.11%	5.60%	2.15%	3.83%

Robust t -statistics in parentheses

* significant at 5%; ** significant at 1%

Table 7: Estimation of Realized Market Price of Risk using 1-year Window

Table 7 reports the analogous parameter estimates using realized returns for all four models. The realized returns were calculated over a 1-year window, and the market price of risk was inferred using the implied volatility from VIX and the 1-year Treasury Bill. The realized market price of risk was then regressed on $x_t = \text{S\&P}_t/\text{S\&P}_{t-5,t-6}$. All independent variables are calculated in the same fashion as those in the expected market price of risk regression. For the calculation of β , $r_L = r_{10}$. t -stats are listed in parenthesis.

$$\frac{(\text{S\&P}_{t+1} + D)/\text{S\&P}_t - 1 - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + e_t$$

$$\frac{(\text{S\&P}_{t+1} + D)/\text{S\&P}_t - 1 - r_{1t}}{\text{VIX}_t} = \lambda_0 + \lambda_1 x_t + \beta \frac{r_{L,t} - r_{1t}}{\text{VIX}_t} + e_t$$

$$\frac{\text{S\&P}_{t+1} + D}{\text{S\&P}_t} - 1 - r_{1t} = a_0 + a_1 \text{VIX}_t + a_2 \text{VIX}_t x_t + a_3 x_t + \beta (r_L - r_S) + e_t$$

	Model 3.1 & 3.2		Model 3.3				Model 3.4	
			1st Stage	2nd Stage	1st Stage	2nd Stage	1st Stage	2nd Stage
x_t	-0.61 (33.61)**	0.96 (11.48)**	-0.90 (35.43)**	-0.90 (46.47)**	1.05 (11.17)**	1.05 (11.04)**		
x_t^2		-0.39 (18.26)**			-0.49 (19.62)**	-0.49 (19.87)**		
λ_0	1.59 (43.29)**	0.13 (1.70)	2.59 (40.06)**	2.59 (68.64)**	0.84 (9.88)**	0.84 (10.43)**		
$\lambda_0(1-w)\sigma$							0.52 (23.92)**	0.53 (29.62)**
$\lambda_0 w$							0.04 (0.52)	0.04 (0.54)
$\lambda_1 w$							-0.02 (0.52)	-0.02 (0.54)
$\lambda_1(1-w)\sigma$							-0.18 (22.39)**	-0.18 (26.58)**
β			-5.01 (24.70)**		-5.24 (26.62)**		-5.61 (23.90)**	
Obs	4678	4678	3560	3560	3560	3560	3560	3560
R^2	14.79%	17.79%	19.73%	31.29%	24.70%	36.39%	22.77%	37.35%

Robust t -statistics in parentheses
 * significant at 5%; ** significant at 1%

Table 8: Realized versus Expected Returns

Table 8 reports the realized and expected returns over the time period. The realized returns, R_t , are calculated as the annualized 30-day return, $R_t^m = \left[\left(\text{S\&P}_{t+1/12} + D_{t+1/12} \right) / \text{S\&P}_t \right]^{12} - 1$. The -327.2% annualized loss was over the 9/23/87-10/26/87 period, where the index fell 28.5%, from 319.72 to 227.42. The expected returns, $E(R_t)$ are the model-dependent returns derived from the dividend-growth model $(D_{0t}/P_t)(1 + g_{1t}) + g_{1t} - r_{1t}$ as given in Section 3.1. The *estimated* realized returns, \widetilde{R}_t , and *estimated* expected returns, $E(\widetilde{R}_t)$, are the predicted values derived from coefficient estimates from the regression of eq. (5).

Variable	Obs	Mean	SD	Min	MAX
R_t	4795	11.91%	53.08%	-327.2%	238.9%
\widetilde{R}_t	4795	12.39%	6.83%	-8.3%	40.6%
$E(R_t)$	4795	7.97%	1.68%	4.0%	11.4%
$E(\widetilde{R}_t)$	4795	7.96%	2.08%	4.5%	20.2%

Table 9: Estimation of Realized and Expected Market Price of Risk with Heteroscedasticity Correction

Table 9 reports the parameter estimates of the heteroscedasticity corrected OLS estimation of eq. (21) using realized and expected returns. The algorithm for correcting the measurement error in the growth rate, g_t , can be found in Appendix A. The results for all three proxies of wealth are reported. There is no significant statistical difference between these estimates, and those found in Table 2. The results for quadratic regression are available upon request. t -stats are listed in parentheses.

$$\frac{E(\hat{\lambda}_t)}{\hat{\sigma}_{t,\epsilon}} \equiv \frac{[(D_{0t}/P_t)(1 + g_{1t}) + g_{1t} - r_{1t}]/VIX_t}{\hat{\sigma}_{t,\epsilon}} = \lambda_0 \frac{1}{\hat{\sigma}_{t,\epsilon}} + \lambda_1 \frac{x_t}{\hat{\sigma}_{t,\epsilon}} + e_t$$

$$\frac{\lambda_t}{\hat{\sigma}_{t,\epsilon}} \equiv \frac{[(S\&P_{t+1} + D)/S\&P_t - 1 - r_{1t}]/VIX_t}{\hat{\sigma}_{t,\epsilon}} = \lambda_0 \frac{1}{\hat{\sigma}_{t,\epsilon}} + \lambda_1 \frac{x_t}{\hat{\sigma}_{t,\epsilon}} + e_t$$

	Model 1: $x_t = \frac{S\&P_t}{S\&P_{t-5}}$		Model 2: $x_t = \frac{S\&P_t}{S\&P_{t-6}}$		Model 3: $x_t = \frac{S\&P_t}{S\&P_{t-5,t-6}}$	
	$E(\hat{\lambda}_t)$	λ_t	$E(\hat{\lambda}_t)$	λ_t	$E(\hat{\lambda}_t)$	λ_t
λ_1	-0.128 (64.23)**	-0.373 (5.51)**	-0.143 (77.58)**	-0.723 (10.52)**	-0.139 (73.83)**	-0.634 (9.27)**
λ_0	0.373 (98.54)**	1.082 (8.49)**	0.432 (110.84)**	1.871 (13.02)**	0.406 (108.25)**	1.617 (11.95)**
Obs	4795	4795	4795	4795	4795	4795
R^2	81.2%	3.23%	83.7%	4.72%	82.9%	4.04%

Absolute t -statistics in parentheses

* significant at 5%; ** significant at 1%

Figure 1: Sharpe Ratios using the Short-Term Model 3.1

Figure 1 demonstrates the relationship over January 1986 to December 2004 between the Livingston/Philadelphia Fed Sharpe ratio, $[(D_{0t}/P_t)(1 + g_t) + g_t - r_t]/VIX_t$, and the regression-predicted Sharpe ratio, $\lambda_0 + \lambda_1 x_t = 0.517 - .254x_t$, using coefficient estimates from regression eq. (5). The control variable in this case is $x_t = \frac{S\&P_t}{S\&P_{t-5}}$.

$$\begin{aligned} \text{Livingston/Philadelphia Fed Sharpe Ratio} &= \frac{(D_{0t}/P_t)(1 + g_t) + g_t - r_t}{VIX_t} \\ \text{Regression-predicted Sharpe Ratio} &= 0.517 - .254x_t \end{aligned}$$

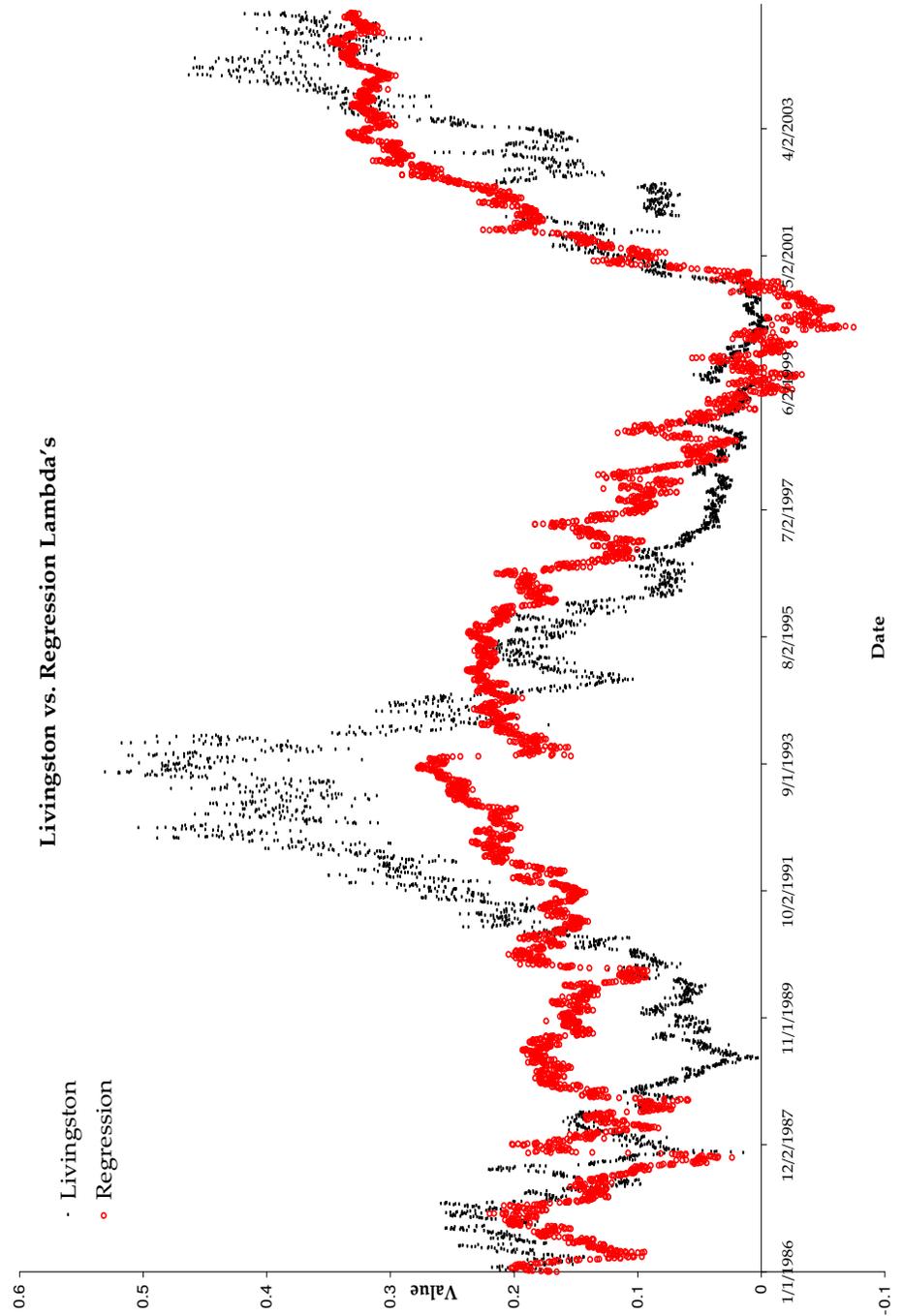


Figure 2: Short- and Long-Term Expected Risk Premiums

Figure 2 demonstrates the relationship over January 1986 to December 2004 for the predicted values of the expected risk premium using the four models. (Models 3.1 and 3.2 differ only in the parameter values of the estimated λ 's, as a consequence of the two models' estimation uses in 3.2 or neglects in 3.1 the *two*-growth rate model.) For all models, $x_t \equiv \frac{\text{S\&P}_t}{\text{S\&P}_{t-5, t-6}}$.

Models 3.1 and 3.2: $\mu_t = r_{1t} + (\lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2) \text{VIX}_t$

Models 3.3: $\mu_t = r_{1t} + \beta (r_{Lt} - r_{1t}) + (\lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2) \text{VIX}_t$

Models 3.4: $\mu_t = r_{1t} + \hat{\beta} (r_{Lt} - r_{1t}) + a_0 + a_1 \text{VIX}_t + a_2 \text{VIX}_t x_t + a_3 x_t$

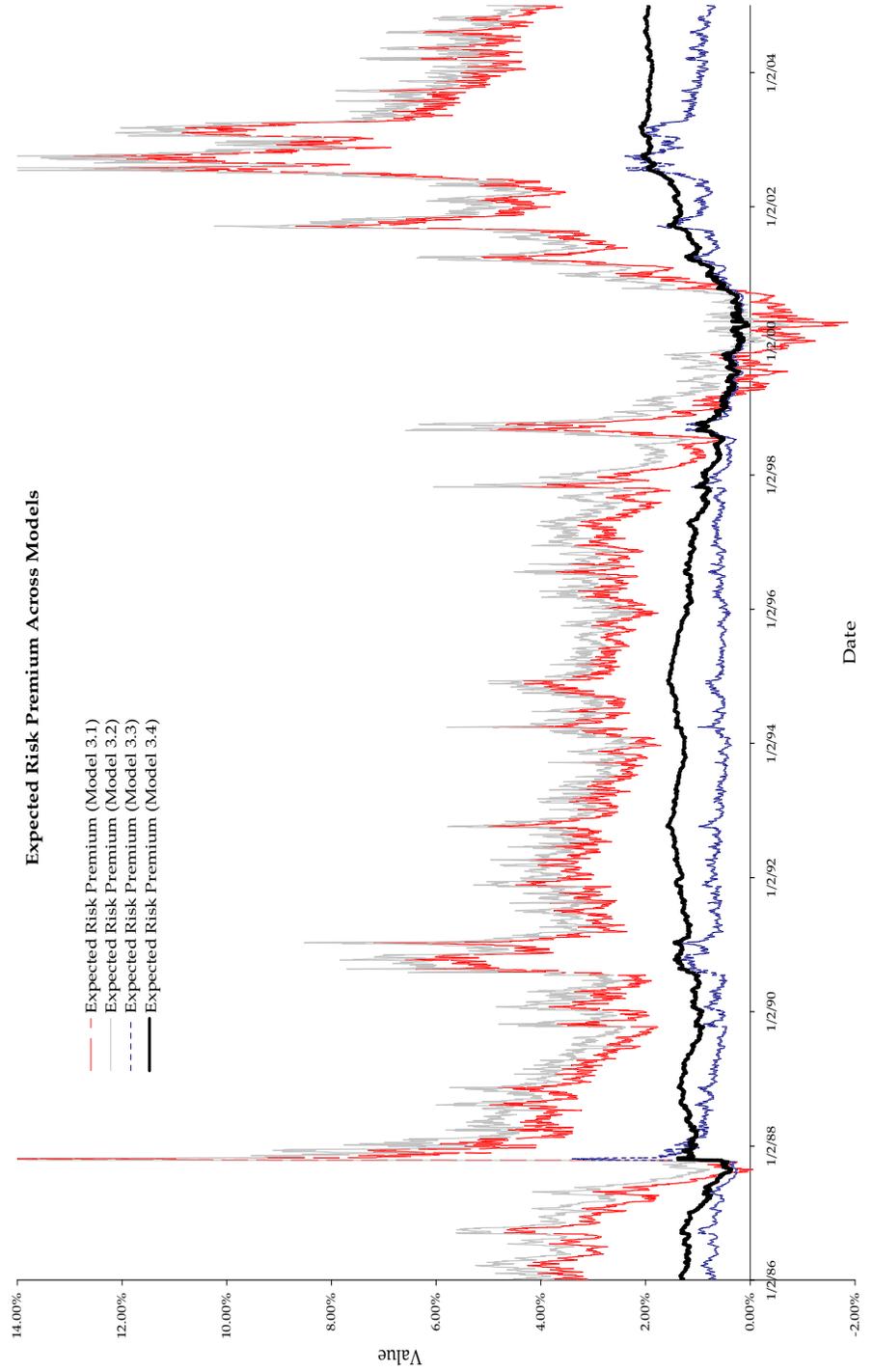


Figure 3: Expected Risk Premia using Model 3.4

Figure 3 demonstrates the relationship over January 1990 to December 2004 between the Livingston/Philadelphia Fed expected risk premia, $(D_{0t}/P_t)(1 + g_{1t}) + g_{10,t} - \hat{\beta}(r_L - r_S)$, and the regression-predicted expected risk premia, $0.0257 + 0.0094 \text{VIX}_t - 0.0028 \text{VIX}_t x_t - 0.0077 x_t$, using coefficient estimates from regression eq. (17). The control variable in this case is $x_t = \frac{\text{S\&P}_t}{\text{S\&P}_{t-5, t-6}}$.

$$\text{Livingston/Philadelphia Fed Sharpe Ratio} = \frac{D_{0t}(1 + g_{1t})}{P_t} + g_{10,t} - r_{1t} - \hat{\beta}(r_L - r_S)$$

$$\text{Regression-predicted Sharpe Ratio} = 0.0257 + 0.0094 \text{VIX}_t - 0.0028 \text{VIX}_t x_t - 0.0077 x_t$$

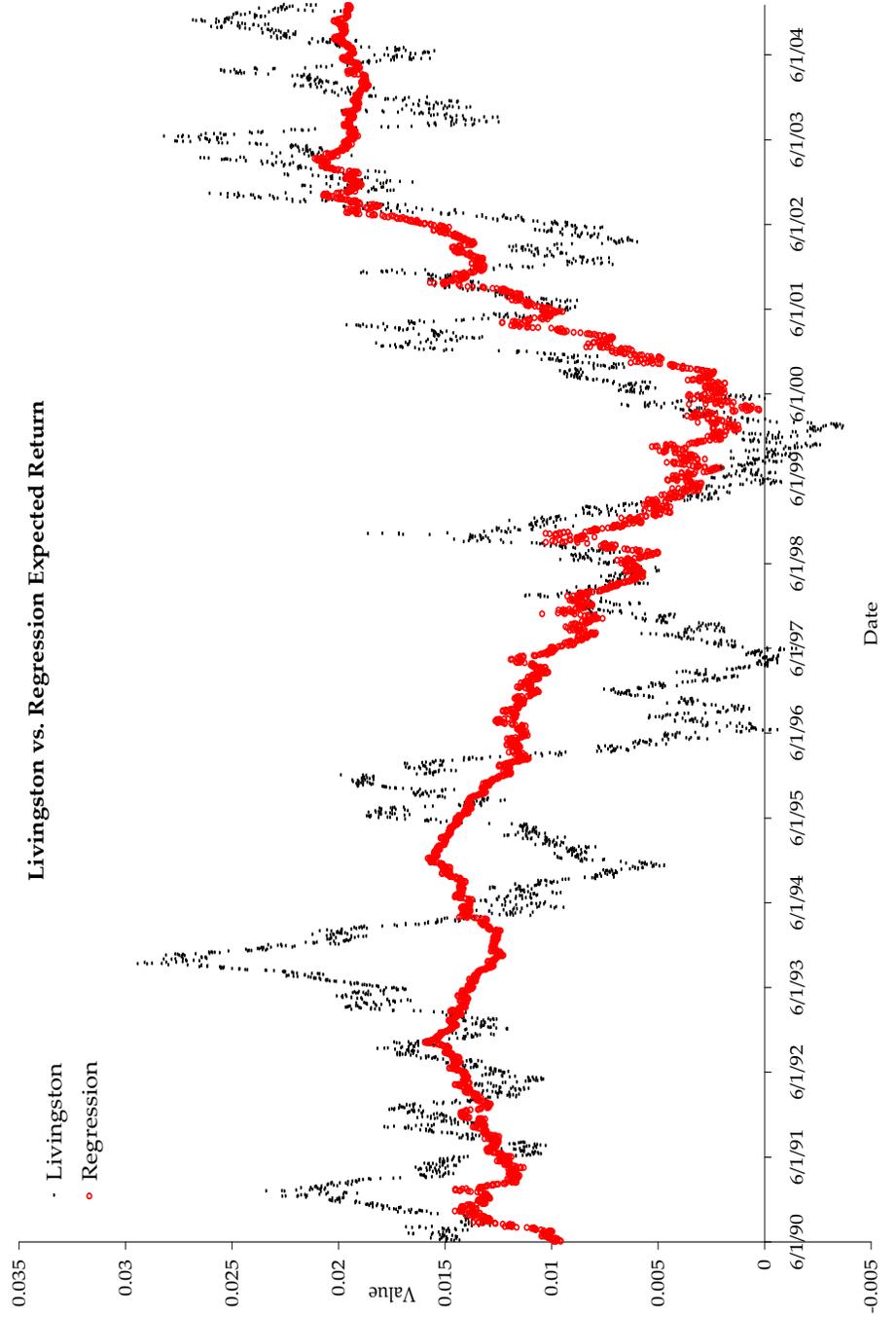


Figure 4: Components for the Long-Term Expected Rate of Return

Figure 4 demonstrates the relationship over January 1986 to December 2004 for the fitted values of the expected risk premium from Model 3.4. The total expected rate of return is broken down into the expected risk premium, and the blended risk-free rate component:

$$\begin{aligned} \text{Blended Risk-Free Rate} &= r_1 + 1.157(r_{30} - r_1) \\ \text{Expected Risk Premium} &= 0.0257 + 0.0094 \text{VIX}_t - 0.0028 \text{VIX}_t x_t - 0.0077 x_t \end{aligned}$$

