Identifying Non-Cooperative Behavior Among Spouses: Child Outcomes in Migrant-Sending Households

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Abstract
In the presence of asymmetric information, allocations can only be coordinated to the extent that each can be monitored, and household decision-making may not be fully cooperative. Because this information problem is particularly acute when individuals are not co-resident, I examine households in which the father migrates without his spouse and children. Results from the China Health and Nutrition Survey indicate that, when the father is away, girls' household labor increases while mothers' total work hours decrease. This is inconsistent with a unitary model in which there is no non-cooperative behavior and household members simply reallocate time to compensate for the father's absence. Furthermore, outcomes that are easily observed by the father - child schooling and health - are not affected by migration, controlling for changes in income. This is also inconsistent with a non-unitary model in which mothers' bargaining power increases when fathers migrate, given existing evidence which suggests that mothers have stronger preferences than fathers for these goods. I propose a simple game-theoretic model of bargaining under asymmetric information and argue that this is consistent with the data. Additional implications are then tested regarding how and which allocations are likely to be affected.

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I. Introduction.

Economic studies of households have increasingly moved away from a unitary model and toward a collective model in which expenditures are determined through some bargaining process. In this collective framework, decision-makers within a household may have divergent preferences, and thus both the monetary value and the ownership of income streams will be important. Given that household members bargain over decisions and that control over resources affects the allocation of those resources, it is natural to consider whether and how individuals may behave strategically in order to increase their own utility. I examine an information problem that permits an individual to conceal expenditures and/or allocations from his/her spouse. This is a weaker form of non-cooperative behavior in that intra-household allocations are only coordinated to the extent that they can be monitored. Migration presents a clear opportunity for such behavior. The migrant has limited ability to observe expenditure and allocation decisions made by the spouse remaining at home but may also be able to conceal his own expenditures by determining the amount of money that will be remitted to the household.

Identifying this type of non-cooperation among spouses will provide additional information about how men's and women's preferences differ, as well as the extent to which the transparency of income matters for the distribution of resources among household members. The existence of such behavior among household members would suggest that expanding opportunities for migration will have different effects on expenditure patterns than simply increasing the amount of income received by the household. Changes in earned income and the potential to earn income will affect bargaining among spouses, but non-cooperative behavior will have an additional effect on the final distribution of expenditures and allocations. The economic literature on the impact of remittances on migrant-sending households (*e.g.* Yang, 2004;

Edwards and Ureta, 2003) has largely neglected a key feature of such income, *i.e.* the difficulty inherent in monitoring the disbursement and allocation of remittances (for an exception, see Chami *et. al.*, 2003). With the rising trends in both rural-urban and international migration, it is essential to understand this dynamic of household decision-making in order to assess the ultimate impact on sending families, child welfare and gender disparities. Non-cooperative behavior would also have important implications for policy and program design because it implies that the channel through which income is received can have important spillover effects, even beyond any direct effect on income or bargaining power. For example, direct subsidies are easily observed by other household members, whereas micro-credit loans and the proceeds of micro-credit enterprises could be concealed from one's spouse and used to finance expenditures that otherwise would not be undertaken.

I introduce incomplete information into a model of household decision-making such that spouses do not observe all actions taken by their partner and also may not be able to deduce the actions from observable outcomes. In this case, the transaction costs associated with enforcing a cooperative arrangement can be prohibitively high such that cooperative and non-cooperative equilibria cannot co-exist. However, when non-cooperative behavior is detected with some positive probability, the optimal allocations will be responsive to the degree of observability. That is, goods which are more difficult to monitor should exhibit larger changes. This is distinct from a change in bargaining power, which would lead to a shift in consumption towards all goods the individual prefers more than does his/her spouse. Non-cooperative behavior may occur on either an extensive or intensive margin. The former reduces the pool of resources over which bargaining occurs, whereas the latter circumvents the distribution of resources which was initially agreed upon via the bargaining process. This paper will focus on non-cooperative behavior that occurs on the intensive margin, *i.e.* with respect to the allocation of resources among goods and individuals.

Data are drawn from the China Health and Nutrition Survey. Both the unobservable time- and child-invariant characteristics of the household can be accounted for with the panel aspect of these data, and controls for household full income are included to differentiate any non-cooperative behavior from income effects. Results indicate that wives of migrant workers are, in fact, engaging in non-cooperative behavior, in a somewhat surprising way. Observable outcomes such as children's school enrollment, schooling attainment and anthropometric measures exhibit no significant change with a change in the father's migrant status. The stability of these outcomes is inconsistent with a case in which migration increases mother's bargaining power, given existing evidence that mothers have stronger preferences than fathers for these goods (see Qian, 2005; Duflo, 2003 and Thomas, 1990). However, unobservable allocations such as nutritional intake and time spent in household chores do change. Because migration of the father also reduces the total time available for household production, these results alone cannot confirm the presence of non-cooperative behavior. Extending the analysis to mothers, I find that their time in both household and labor market activities falls when the husband is away. The increase in mothers' leisure is inconsistent with a model in which there is no non-cooperative behavior and all household members simply reallocate time to compensate for the father's absence.

The following section will situate this paper in the existing literature on non-cooperative behavior within households. Section III discusses the implications of migration in standard models of the household, both unitary and non-unitary. Section IV discusses the empirical implementation and shows that the data are inconsistent with cooperative models of the household. In Section V, I present an alternative model of bargaining under asymmetric information and argue that this is consistent with the data. Additional implications of this model are tested in Section VI, and Section VII concludes.

II. Non-Cooperative Decision-Making

Within a collective household model, individuals may be either cooperative or non-cooperative. Cooperation implies that household members can negotiate and then commit to binding and costlessly enforceable agreements. The starting point for negotiation is each individual's "threat point", the maximum utility he/she could expect to attain in the absence of a cooperative agreement. Manser and Brown (1980) and McElroy and Horney (1981) take divorce as the threat point. Any circumstances that affect the individual's welfare upon dissolution of the marriage (e.g. value of personal assets, labor market opportunities, divorce law, transfers, etc.) will also affect the threat point and, consequently, his/her bargaining power within the marriage. Alternatively, Lundberg and Pollak (1993) propose a non-cooperative equilibrium as the threat point, in which each spouse maximizes his/her own welfare, given the behavior of his/her spouse. In the non-cooperative equilibrium, the marriage remains intact, but the individuals do not coordinate their actions or pool their resources. This is a more plausible model of household bargaining when divorce is costly, both in monetary and emotional terms. The non-cooperative equilibrium may also be preferred over the cooperative equilibrium when the transaction costs of negotiating, monitoring and enforcing cooperative arrangements are very high or when the gains from cooperation are relatively low.

Household public goods are the distinguishing feature of a household, whether members are cooperative or non-cooperative. When the potential contributors to a public good each make strictly positive contributions, control over resources will not affect the equilibrium level of the public good or the equilibrium utilities of the individuals, even if the individuals do not coordinate (Warr, 1983 and Bergstrom, 1986 as cited in Lundberg and Pollak, 1993). However, if the provision of household public goods is organized along "separate spheres", *i.e.* there is specialization by gender, such that one or both spouses make zero contributions to some public good, control over resources will affect the equilibrium outcome. This is true even when control over resources does not affect the utilities that individuals could obtain outside of the marriage, as in the case of a child allowance which is provided to either married mothers or married fathers but is always provided to mothers upon divorce. Uncertainty about income realizations and hence threat points does not affect the basic result, but the authors do not consider the case where income realizations and/or allocation decisions are not perfectly observable by both household members. Dubois and Ligon (2004) introduce asymmetric information into a unitary model of the household and examine which factors determine the intra-household allocation of calories. They reject the hypothesis that food is efficiently allocated among household members and find suggestive evidence that it is, instead, allocated both to create incentives for individuals and as a form of nutritional investment.

These models are taken as the starting point for this paper. I consider a weaker form of non-cooperative behavior in which household members coordinate allocations only to the extent that they can be contracted on, *i.e.* monitored and verified at reasonable cost. When the asymmetric information is such that certain allocations are not fully contractible, spouses will attempt to deviate from the cooperative outcome, but deviations will be constrained by the possibility of detection. Recent evidence from Ashraf (2004) confirms that individuals do attempt to conceal expenditures from their spouses when presented with the opportunity. The prevalence of such non-cooperative behavior in an experimental setting suggests that this is an important phenomenon to consider in a more general context. de Laat (2005) finds evidence that migrants living in Nairobi invest in costly monitoring technologies to mitigate moral hazard on the part of their spouses in rural villages, but increased monitoring has little effect on their wives' behavior. This paper focuses, instead, on outcomes in the migrant-sending household and shows that allocations are, in fact, responsive to the efficacy of monitoring.

III. Implications of Migration in Cooperative Households

In the absence of non-cooperative behavior, the effect of migration on intra-household allocation consists of three components: a reduction in the amount of time available for household production, an increase in household income and, in a non-unitary model, a change in the distribution of bargaining power between spouses. The appropriate counterfactual for identifying non-cooperative behavior is, then, the set of allocations that would be chosen by the household, conditional on these changes to bargaining power and the time-budget constraint, if both spouses could costlessly commit to cooperation. Thus, before considering the possibility of non-cooperation, the implications of migration in standard cooperative models of the household, both unitary and non-unitary, must first be established.

Unitary Model

A household consists of two adults, a migrant (m) and a non-migrant (n), and one child (k). The household has preferences over adult private consumption (x) and a household public good (z). Production of the household good depends on the time contributed by each individual, as well as person-specific productivities (τ). Adults may engage in labor market activities that earn a wage of w per unit of time, and children cannot participate in income-generating activities but can assist with household production. Rather than specifying a utility of leisure, I allow time spent in productive activities to provide some disutility.

$$\max\ U(x_m, x_n, z, t_m, t_n, t_k) \tag{1}$$

where $x_m + x_n = w_m t_m^w + w_n t_n^w$, $t_i = t_i^w + \mu t_i^h$ for $i = m, n$ and $z = z(t_m^h, t_n^h, t_k; \tau_m, \tau_n, \tau_k)$

Because household and wage labor may have differing effects on utility, the time spent in household production is scaled by a parameter μ in the utility function which reflects the disutility of household labor relative to wage labor. Total private consumption must be equal to total earnings; there is no savings and no borrowing.

Comparative statics (see Technical Appendix, Section A for derivation) indicate that a reduction in father's household labor, holding wages constant, increases mother's time in household production and has an ambiguous effect on child labor. This is because, for a compensated increase in wages, fathers increase market labor supply and reduce household labor supply. With a utility function concave in x and z, total household utility can be increased by reallocating mother's labor from the wage sector to the household. The effect on child labor is ambiguous because an increase in child labor has both a direct negative effect on household utility and an indirect positive effect through the increase in z. The income effect of an increase in father's wages, holding father's household labor fixed, increases mother's time in household production and reduces child labor.

Non-Unitary Model

Next, I allow the adults in the household to have different preferences and assume that household members can negotiate binding agreements with zero transaction costs, *i.e.* decision-making is fully cooperative. In this case, the household maximizes a weighted sum of the individual utility functions.

$$\max \ \lambda U_m(x_m, z, t_m, t_k) + (1-\lambda) U_n(x_n, z, t_n, t_k) \qquad [2]$$

$$\text{where } x_m + x_n = w_m t_m^w + w_n t_n^w,\ t_m = t_m^h + \mu_m t_m^w,\ t_n = t_n^h + \mu_n t_n^w,\ z = z(t_m^h, t_n^h, t_k; \tau_m, \tau_n, \tau_k)$$

The bargaining weights (λ,1 – λ) are a function of the individuals' outside options, θ_m and θ_n, respectively. Changes in fathers' wages and the time available for household production have the same effects on household labor supply in both the unitary and non-unitary models, although the magnitude of the effects differs (see Technical Appendix, Section B). However, in the non-unitary model, migration may also affect intra-household allocation via a change in the distribution of bargaining power between spouses. The change in bargaining power is ambiguous; husbands have an increase in wages, but wives must bear a larger burden in household production when their husbands are away. Because non-resident husbands must rely on their wives for provision of the public good, migration may increase θ_n relative to θ_m. Comparative statics (see Technical Appendix, Section B) show that an increase in father's bargaining power, holding his wage and household labor fixed, increases mother's time in household production and reduces child labor. Finally, an increase in father's bargaining power reduces mother's private consumption.

To summarize, the unitary model predicts that mothers' household labor should increase when fathers migrate, and child labor may either increase or decrease, depending on the extent to which children are required substitute for fathers in household production. The cooperative non-unitary model predicts the same, except when migration increases mothers' bargaining power. These implications are sensitive to the restrictive assumption that all goods are separable in utility. The following section examines whether these predictions are consistent with the data and considers whether relaxing the separability assumption could generate the observed pattern.

IV. Empirical Application

This section first provides descriptions of the data and empirical specification. I then present the main results and argue that the findings are inconsistent with standard cooperative models of the household. A robustness check is also presented to determine if data limitations may be driving the results.

Data and Background

Data are drawn from the China Health and Nutrition Survey (CHNS). The sample includes roughly 4,000 households, approximately 15,000 individuals, drawn from nine diverse provinces. Households were first surveyed in 1989, with follow-ups in 1991, 1993, 1997 and 2000. Attrition at the household level is less than 5% between waves, and replacement households were added in 1997 and 2000. The timing of the survey is well-suited for the study of migration, as the 1990s were a period of rapid growth in intra-national labor migration. Using population surveys, Liang and Ma (2004) find that the number of inter-county migrants in China increased from 20 million in 1990 to 45 million in 1995 and 79 million in 2000. This was, in large part, due to a relaxation of migration restrictions in 1988, which allowed individuals to obtain legal temporary residence in other localities. Increased openness and marketization in the 1990s also spurred economic growth, which increased the demand for construction and service workers in urban areas (de Brauw and Giles, 2005).

The panel nature of the data allows for inclusion of individual fixed effects to account for the endogeneity of migration, and community-year fixed effects are included to control for time-varying factors. Migrants are defined as individuals living away from the household for at least one full month in the previous year. The sample of migrant-sending households is further limited to those in which the father was away from the household for all seven days in the week prior to enumeration, because most outcomes of interest are defined over the previous week. Descriptive statistics are presented in Tables 1 through 3, with observations at the person-year or household-year level. Differences in observable characteristics between migrant and non-migrant households are relatively minor. Children in migrant households are somewhat more likely to be enrolled in school, and also more likely be engaged in household labor. The migrants themselves appear to be positively selected on schooling, as are their spouses. Migrant households hold less value in productive assets and have higher household income, on average.

Empirical Specification

I estimate reduced-form demand equations for children's schooling and health and household labor of both children and mothers. Data on the quantity of time spent in various household activities was collected inconsistently across surveys, so identification must rest on changes in household labor on the extensive rather than intensive margins. For individual *i* in household *j* in community *a* at time *t*, the demand for good *y* can be expressed as

$$y_{ijkt} = \alpha + \beta \cdot h_{jkt} + \phi \cdot c_{ijkt} + \delta \cdot away_{jkt} + \gamma \cdot (d_{ijkt} \cdot away_{jkt}) + \rho \cdot (months\ away_{jkt}) + \upsilon_{ij} + \eta_{kt} + \pi_t + \varepsilon_{ijt}$$

where *h* is a vector of time-varying household characteristics, *c* is a vector of individual covariates, and *d* is a subset of those covariates which are allowed to vary with father's migration status. The error term consists of four components – an individual effect that is fixed over time (υ), a time effect that is specific to the community of residence but fixed across individuals within the area (π), a general time trend (η), and a mean-zero i.i.d. disturbance (ε). All specifications include community-year fixed effects and person-specific fixed effects to control for unobserved characteristics that are correlated with the migration decision. Controls for the father and mother's current wages are included to account for changes in household income over time. For individuals engaged in occupations that do not pay by time or piece rate, predominantly agricultural work, the wage is imputed as the prevailing daily wage for an unskilled farm laborer. Additional control variables include a quadratic in age, parents' ages (for child-level regressions), assets owned (farm land, farming equipment, value of small business capital and area of owned home), household size, number of children (number of siblings for child-level regressions), the sex composition of children (siblings), as well as month of survey. Parents' schooling attainment changes very little over time and is therefore subsumed into the fixed effect.

A quadratic in the months the father is away is included for two reasons. First, investments in human capital and the allocation of household labor likely require some time to adjust. That is, measures of health (body mass index, skin fold, arm circumference) reflect prior investments and do not necessarily adjust instantaneously to changes in inputs. For household tasks that require learning by doing, there may be fixed costs involved with reallocating labor. Second, the wage variables reflect labor market opportunities available at the time of the survey. If migrants earn higher wages only while living away from home, including measures of the duration of migration episodes will provide better controls for changes in total household incomes. Age and the number of siblings, by gender, are allowed to vary with father's migrant status because these characteristics affect demand for the child's household labor. The effects differ with migration status because fathers cannot contribute to household production when they are not co-resident. Returns on investments in schooling and health are also likely to vary with age, in which case remittance income may not be allocated identically to children of varying ages. Furthermore, children in larger families receive a smaller share of household resources, implying that the income effect of migration varies with the number of siblings.

Basic Results

Columns I and II of Table 4 present the child-fixed effects estimates of the relative effect of migration on household labor for children aged 6 to 16. The probability that daughters do any household chores (purchasing food, preparing food, laundry) is increasing in the number of months the father is away, and the opposite is true for sons. This pattern suggests that children, particularly younger children, are required to do more household chores increases when fathers are away for a sufficiently long period of time (>5 months). It is difficult to obtain more precise estimates with such a coarse measure of household labor, and data on actual hours were collected inconsistently across waves. Estimates for specific chores (see Table 8) provide more conclusive evidence that children's household labor increases when fathers are away; these results will be discussed in more detail in Section VI. The changes in chores also do not appear to be offset by changes in other tasks. I find no significant effects of migration on the probability that children engage in non-wage labor activities such as gardening, household farming, livestock care, fishing, or handicrafts. The point estimates are quite small in magnitude and generally smaller than the point estimates for chores. An increase in child household labor would be consistent with a standard unitary model of the household in which individuals must reallocate time in order to compensate for the father's absence.

The unitary model also predicts that mothers should unambiguously increase time in household activities when fathers migrate. Columns I and II of Table 5 present estimates of the effect of migration on mothers' time allocation. The probability that mothers do any of the enumerated household chores (purchasing food, preparing food, laundry) is decreasing in months the father is away, although the point estimate is not statistically significant. Again, estimates for specific chores, presented in Table 9, provide more conclusive evidence that mothers spend less time in household maintenance when fathers are away. Furthermore, the number of months the father is away has a significant negative effect on the total time mothers spend in income-generating activities (wage labor plus "other" non-wage work activities such as gardening, household farming, livestock care, fishing, or handicrafts). When taken together, these results suggest that mothers are consuming more leisure. An increase in mothers' leisure is inconsistent with migration in a unitary model of the household, and it is difficult to imagine a pattern of complementarities in utility or production that could generate both a reduction in mothers' labor and an increase in child labor. If private and public goods are strong complements in utility, an increase in income and reduction in time available for household production could reduce the demand for public goods. However, because child labor provides direct disutility, this complementarity could not produce an increase in child household labor without an increase in mothers' time in income-generating activities.

Complete time diaries are not available in the CHNS, making it difficult to conclude that the observed reduction in mothers' work hours signifies an increase in mothers' leisure. It is possible that fathers engage in other household activities which are not enumerated and migration forces mothers to substitute into these tasks while children substitute for mothers in the enumerated household tasks. To investigate this possibility, I utilize an alternate sample of households in which the father experiences an illness or injury sometime in the four weeks prior to the survey date. The number of days a health complaint disrupted the individual's normal activities is used to measure the extent to which time available for household production was affected, and both individual and community-year fixed effects are again included.

If the results presented above are driven by increased demand for mothers' time in activities typically carried out by fathers, this alternate sample should yield similar findings.

That is, if wives of migrants reduce time in laundry, food preparation and food purchase in order to substitute for husbands' time in other, non-enumerated activities, this same reduction should be evident when husbands' household labor is reduced by illness or injury. Estimates in Table 6 indicate that this is not the case; the probability that mothers do any of these three chores is largely unaffected when fathers experience a debilitating illness or injury, and the point estimates are, in fact, positive. However, the number of days that fathers are debilitated by illness or injury is relatively short, on average. The sample mean is 13 days, and roughly 45% of fathers are debilitated for less than one week. It is possible that households simply do not adjust time allocation in such short periods. This hypothesis is not supported by the estimates in the child-level regression. Younger sons and older daughters are more likely to be engaged in some form of household labor when fathers are debilitated (estimates for specific household chores are similar and not presented here). Thus, the estimated effect of migration on mothers' and children's time allocation cannot be fully explained by the existence of other non-enumerated tasks which are typically carried out by the father.

Under a non-unitary model of the household, an increase in mother's bargaining power could explain an increase in child household labor accompanied by a decrease in mother's household labor. However, an increase in mothers' bargaining power should also be accompanied by changes in other goods favored by the mother, *e.g.* children's human capital (see Duflo, 2003; Thomas, 1990; Qian, 2005), irrespective of the ease with which those goods can be monitored. Columns III and IV of Table 4 present the child-fixed effects estimates of the relative effect of migration on easily observable outcomes for children aged 6 to 16. Migration of the father has no statistically significant effects on school enrollment, and the average marginal effect is quite small. Similarly, migration has no statistically significant effects on children's body mass index, and the absolute effects implied by the point estimates are relatively small. For a 4-foot tall child with BMI in the normal range (weighing 60-80 pounds), a half point change in BMI is equivalent to change in weight of approximately 2 pounds.

Given findings in other studies, the observed stability in schooling and health for both boys and girls appears inconsistent with a model in which mothers' bargaining power increases when fathers migrate. Qian (2005) finds that, among agricultural households in China, an exogenous increase in the share of female income has a significant positive effect on educational attainment for all children, whereas increasing the share of male income reduces educational attainment for girls. Chen (2005) finds that girls' school enrollment increases relative to boys when mothers have increased bargaining power, and Duflo (2003) and Thomas (1990) find that an increase in female income improves health outcome for all children and has a disproportionately positive effect on girls. Finally, the third column of Table 5 indicates that migration of the father does not improve mothers' health, which again appears inconsistent with an increase in mothers' bargaining power.

V. Migration with Imperfect Information

The results presented above cannot be easily explained by standard cooperative models of the household, either unitary or non-unitary. I argue that this is because migration introduces imperfect monitoring – the migrant has limited ability to observe allocation decisions made in his absence, and the spouse remaining at home may not be able to observe the wages or expenditures of the migrant. Imperfect information increases transaction costs associated with enforcing cooperative bargaining agreements and thus reduces the gains from cooperation. It also affects the utility associated with non-cooperation, as each spouse can only react to the allocations that are observed, not necessarily those that actually occurred. Thus, an individual who chooses not to cooperate will not always receive a commensurate response from his/her spouse.

In this section, I model household decision-making as an infinitely repeated game, allowing for asymmetric information. I will first describe an equilibrium of this game in which the migrant cooperates and his spouse does not and define the conditions under which such an equilibrium is feasible. I then discuss the actions consistent with the equilibrium strategies and how these actions differ from the cooperative allocations that could be obtained in the absence of imperfect information. Finally, testable implications are derived by examining how the optimal non-cooperative strategy varies with the parameters of the model. Because the CHNS provides data only on sending households, I focus on the case in which there is imperfect monitoring of allocations under the mother's control and assume that there is perfect information regarding the earnings and expenditures of the migrant. A more complete dynamic model in which wives have beliefs about husbands' wage realizations and update those beliefs in each period is left to future research. Furthermore, imperfect monitoring of the migrant's actions would not obviate non-cooperative behavior on the part of his spouse, provided that asymmetric information prevents the couple from attaining a fully cooperative equilibrium.

Description of Game and Equilibrium

Utility and production functions are the same as in the basic non-unitary model described above. Players, a migrant (m) and a non-migrant (n), have preferences over private consumption (x), a household public good (z) that is produced with household members' time, own time spent in productive activities (t^w and t^h), and their child's time spent in household production (t_k). Children cannot participate in income-generating activities but can assist with household production. Adults may engage in labor market activities that earn a wage of w per unit of time, and the migrant makes a transfer (s) to his wife. Before the game begins, *i.e.* before the husband migrates, the couple negotiates a set of cooperative allocations.

Definition. The cooperative allocations under asymmetric information, denoted $\{t_m^{w^c}, s^c, t_n^{w^c}, z^c, t_n^{h^c}, t_k{}^c\}$, maximize $V_m(\sigma_m{}^*, \sigma_n{}^*)$subject to $V_n(\sigma_m{}^*, \sigma_n{}^*) \geq \theta_n + S(1-\lambda, \sigma_m{}^*, \sigma_n{}^*)\Psi$, where $\sigma_m{}^*$ and $\sigma_n{}^*$ are the equilibrium strategies, $V(\cdot)$ denotes the expected present discounted value of the equilibrium payoff, and θ denotes the outside option. $S(\cdot)$ is the sharing rule that determines how surplus utility, $\Psi = (V_m(\sigma_m{}^*, \sigma_n{}^*) - \theta_m) + (V_n(\sigma_m{}^*, \sigma_n{}^*) - \theta_n)$, is allocated.

These allocations are not necessarily equivalent to those that would be obtained in the absence of imperfect information, precisely because the incentive problem may not allow the household to achieve a first-best Pareto optimal outcome. However, conditional on the equilibrium strategies, $\{t_m^{w^c}, s^c, t_n^{w^c}, z^c, t_n^{h^c}, t_k{}^c\}$ is Pareto efficient, *i.e.* the cooperative allocations in the presence of asymmetric information are constrained Pareto optimal. Note that, if both spouses play *cooperate* in equilibrium, the cooperative allocations and equilibrium payoffs are equivalent to those that would be obtained in the absence of imperfect information, provided that no costly monitoring technologies are utilized.

When the father migrates, allocations move into "separate spheres" such that each spouse has direct control over only a subset of goods. This division is determined by residence patterns, *i.e.* the absence of the father from the sending household.

Definition. The migrant's action space is limited to the choice of $t_m^w \in [0,T]$, where T is the time endowment, and a transfer to his wife $s \in [0, w_m t_m^w]$. The mother's action space includes her own market and household labor, $t_n^w \in [0,T]$, $t_n^h \in [0,T]$ and the child's household labor, $t_k \in [0,T]$. The father cannot contribute to household production, and thus the mother's choices of t_n^h and t_k fully determine z. If play enters into a punishment phase, each player determines the length of time (denoted f when player m punishes player n and denoted g when player n punishes player m) to punish his/her spouse.

Any actions not consistent with the cooperative allocations are considered non-cooperative. I restrict attention to actions that are consistent with a Cournot-type strategy, as in Lundberg and Pollak (1993), which effectively limits the action space to two strategies – *cooperate* and *don't cooperate*. The game then proceeds as follows. At the beginning of each period, both players choose the allocations associated with their respective spheres. Payoffs are then realized. With probability one, the wife observes the allocations her husband has chosen while living outside the household. The migrant observes his spouse's actions with probability $q < 1$ and the cooperative allocations with probability $(1 - q)$. The probability of detection depends on the actions of both players as well as a set of exogenous parameters (*e.g.* the ability to observe changes in body mass index versus changes in caloric intake), and both players have complete information regarding the structure of this q-function. The relationship between actions and the probability of detection will be discussed in further detail below.

Repeated games of this type have multiple equilibria. I focus on an equilibrium in which both spouses play a trigger strategy and define the conditions under which non-cooperation can occur. Consider the following strategy profile. The migrant (m) plays *cooperate* unless he observes his spouse playing *don't cooperate*, in which case play moves into a punishment phase. In the punishment phase, the migrant plays *don't cooperate* until *cooperate* is observed for $f \geq 1$ periods, not necessarily consecutive, at which time he again plays *cooperate*. The non-migrant spouse (n) plays *don't cooperate* in each period. If her spouse plays *don't cooperate* in a period in which she is not being punished, play also moves into a punishment phase until the migrant has played *cooperate* for $g \geq 1$ periods, again not necessarily consecutive. Let * denote the equilibrium actions of player n, c denote the cooperative allocations (also the equilibrium actions of player m when not in a punishment phase), and ' denote the actions chosen by player m when punishing player n. Then the total expected payoffs associated with this strategy profile are

$$V_n = U_n(x_n*, z*, t_n*, t_k*) + \sum_{i=1}^{f-1} \delta^i [(1-q*)^i U_n(x_n*, z*, t_n*, t_k*) + (1-(1-q*)^i) U_n(x_n', z*, t_n*, t_k*)]$$
$$+ [\delta^f/(1-\delta)][(1-q*)^f U_n(x_n*, z*, t_n*, t_k*) + (1-(1-q*)^f) U_n(x_n', z*, t_n*, t_k*)]$$

$$V_m = (1-q*) U_m(x_m{}^c, z^c, t_m{}^c, t_k{}^c) + q*U_m(x_m{}^c, z*, t_m{}^c, t_k*) + \sum_{i=1}^{f-1} \delta^i [(1-q*)^{i+1} U_m(x_m{}^c, z^c, t_m{}^c, t_k{}^c)$$
$$+ (1-q*)^i q*U_m(x_m{}^c, z*, t_m{}^c, t_k*) + (1-(1-q*)^i)(1-q*) U_m(x_m', z^c, t_m', t_k{}^c)$$
$$+ (1-(1-q*)^i) q*U_m(x_m', z*, t_m', t_k*)] + [\delta^f/(1-\delta)][(1-q*)^{f+1} U_m(x_m{}^c, z^c, t_m{}^c, t_k{}^c)$$
$$+ (1-q*)^f q*U_m(x_m{}^c, z*, t_m{}^c, t_k*) + (1-(1-q*)^f)(1-q*) U_m(x_m', z^c, t_m', t_k{}^c)$$
$$+ (1-(1-q*)^f) q*U_m(x_m', z*, t_m', t_k*)]$$

where $x_n* = w_n t_n^w * + s^c$, $x_n' = w_n t_n^w * + s'$ and $q*$ is the probability of detection consistent with the equilibrium actions of both players.

> **Proposition 1.** For $q*$ and f sufficiently small and δ sufficiently high, the above strategy profile is an equilibrium of the infinitely repeated game.

Proof. First, consider a one-stage deviation by player n. The choices $\{t_n^w*, t_n^h*, t_k*\}$ are given by player n's Cournot reaction function; thus, conditional on playing *don't cooperate*, there are no other actions that could yield a higher payoff for player n given player m's strategy. The only possible alternative would be for player n to play *cooperate*. If $f = 1$, this will not yield a higher payoff as long as

$$q* < [U_n(x_n^*, z^*, t_n^*, t_k^*) - U_n(x_n^c, z^c, t_n^c, t_k^c)] / \delta[U_n(x_n^*, z^*, t_n^*, t_k^*) - U_m(x_n', z^*, t_n^*, t_k^*)]. \quad [3]$$

As long as a set of profitable deviations exists, it must be the case that $U_n(x_n^*, z^*, t_n^*, t_k^*) > U_n(x_n^c, z^c, t_n^c, t_k^c)$. With $x_n^* > x_n'$, it must also be the case that $U_n(x_n^*, z^*, t_n^*, t_k^*) > U_m(x_n', z^*, t_n^*, t_k^*)$, and there is clearly some value of $q^* > 0$ for which the above condition is satisfied. However, V_n is decreasing in f, and as f increases, the equilibrium value of q^* must decrease, implying that the optimal actions for player n must be closer to the cooperative actions. As $f \to \infty$, $V_n \to U_n(x_n^*, z^*, t_n^*, t_k^*) + [\delta/(1-\delta)]U_n(x_n', z^*, t_n^*, t_k^*)$ and the equilibrium condition becomes

$$[U_n(x_n^*, z^*, t_n^*, t_k^*) - U_n(x_n^c, z^c, t_n^c, t_k^c)] > \delta[U_n(x_n^*, z^*, t_n^*, t_k^*) - U_m(x_n', z^*, t_n^*, t_k^*)].$$

Given that the optimal non-cooperative actions will be very close to the cooperative actions when k is large, this condition cannot hold when $x_n' << x_n^c$ ($s' << s^c$), *i.e.* when player m can impose a sufficiently large punishment in any one period. Thus, non-cooperative behavior can only occur in equilibrium if f is bounded from above, *e.g.* by social norms, such that the maximum credible punishment is insufficient to deter player n from playing the non-cooperative strategy. Next, consider player n's strategy in the punishment phase. When $f = 1$, she receives a higher payoff by playing *don't cooperate* than by deviating to *cooperate*, provided that

$$q^* < [U_n(x_n', z^*, t_n^*, t_k^*) - U_n(x_n', z^c, t_n^c, t_k^c)] \,/\, \delta[U_n(x_n^*, z^*, t_n^*, t_k^*) - U_m(x_n', z^*, t_n^*, t_k^*)].$$

This is equivalent to [3] if utility is separable in x_n but is a more restrictive condition on q^* if x_n and z are complements in utility. As $k \to \infty$, this condition becomes

$$U_n(x_n', z^*, t_n^*, t_k^*) > U_n(x_n', z^c, t_n^c, t_k^c),$$

which must be true if the strategy *don't cooperate* is optimal for player n.

Now, consider a one-stage deviation from this strategy profile by player m. If he plays *don't cooperate*, it is detected with probability one and punished by player n. If $q^* = 0$, a one-stage deviation to *don't cooperate* will not be profitable as long as

$$\sum_{i=1}^{g} \delta^i [U_m(x_m^c, z^c, t_m^c, t_k^c) - U_m(x_m^c, z', t_m^c, t_k')] > U_m(x_m'', z^c, t_m'', t_k^c) - U_m(x_m^c, z^c, t_m^c, t_k^c),$$

where '' denotes player *m*'s optimal non-cooperative actions, and ' denotes the punitive actions. Alternatively, if $q^* = 1$, a one-stage deviation to "don't cooperate" will not be profitable as long as

$$\sum_{i=1}^{g} \delta^i [U_m(x_m', z^*, t_m', t_k^*) - U_m(x_m^c, z', t_m^c, t_k')] + \delta^{g+1}[U_m(x_m', z^*, t_m', t_k^*) - U_m(x_m^c, z^*, t_m^c, t_k^*)$$
$$> U_m(x_m'', z^*, t_m'', t_k^*) - U_m(x_m^c, z^*, t_m^c, t_k^*).$$

Note that, when player *m* is punishing player *n*, he also plays *don't cooperate*, which implies that $x_m' = x_m''$. Then, given $x_m' = x_m'' > x_m^c$ and $z^* > z'$, this condition must hold for δ sufficiently high. The payoff for player *f* is monotonically decreasing in q^*, and therefore no profitable deviation exists for any value of q^*. Finally, both player *n* and player *m* will be willing to play the specified strategy after observing *don't cooperate* from the other player because, after any such history, play moves into a phase in which the non-deviating player is rewarded while the deviating player is punished. Then, for q^* and *f* sufficiently small and δ sufficiently high, it is not possible to have both cooperative and non-cooperative equilibria. That is, a strategy profile in which both spouses play *cooperate* when the other spouse has also played *cooperate* in the previous period and play *don't cooperate* otherwise cannot be an equilibrium. ■

When the probability of detection is sufficiently low, discount rates are sufficiently high, and the maximum credible punishment that player *m* can impose on player *n* is sufficiently small, non-cooperative behavior can occur in equilibrium. Because q^* depends on the actions of both players, the equilibrium condition regarding q^* is, in fact, a constraint on the exogenous parameters that affect the probability of detection. That is, non-cooperation can only occur in equilibrium if these exogenous parameters are such that, when both players follow a Cournot reaction function, the probability of detection is sufficiently low.

Equilibrium Actions and Comparison with Full Information Case

Now that equilibrium payoffs have been defined, I examine the optimal actions associated with these strategies and compare them to the allocations that could be obtained in the absence of imperfect information.

Proposition 2. The non-cooperative strategy can only yield a higher payoff for player n when $t_n^w \geq t_n^{w\,c}$, $z \leq z^c$, $t_n^h \leq t_n^{h\,c}$, and $t_k \geq t_k^c$.

Proof. Recall that the husband receives zero utility from his wife's consumption and would always agree to either an increase in z via an increase in t_n^h and a proportional reduction in t_n^w (x_n) holding t_k constant, or an increase in t_n^h and proportional reductions in t_n^w and t_k holding z constant.[1] Thus, if there exists a set of allocations $\{t_n^w < t_n^{w\,c}, z > z^c, t_n^h > t_n^{h\,c}, t_k < t_k^c\}$ that could increase V_n, those allocations would also increase V_m. But then $\{t_n^{w\,c}, z^c, t_n^{h\,c}, t_k^c\}$ could not be constrained Pareto optimal. ■

Thus, when it is optimal for the non-migrant spouse to play *don't cooperate*, it must be the case that she produces less of the public good, shifts household labor to the child, and shifts her own labor from the household to the market, relative to the cooperative allocations negotiated under asymmetric information.

Based on the above proposition, define q as follows.

Definition. $q = q(x_n, z, t_n^h, t_k; x_n^{\;c}, z^c, t_n^{hc}, t_k^{\;c}, \omega_q, \omega_x, \omega_z, \omega_h, \omega_k)$ is the probability that non-cooperative behavior is detected, where

[1] If the husband's utility depended on his wife's utility, he would still accept such a change as long as the arrangement provides more utility to the wife. Similarly, if the husband received direct utility from x_f and t_f, he would still accept such an arrangement as long as he has stronger preferences for t_c than for x_f and t_f. The only cases in which this claim would not hold are (1) if husbands' disutility from wives' time in household production exceeds the disutility from children's time in household production, or (2) children's time in household production provides fathers with positive utility over some range.

$$\frac{\partial q}{\partial x_n} > 0, \frac{\partial^2 q}{\partial x_n^{\,2}} > 0 \text{ for } x_n > x_n^{\,c}; \ \frac{\partial q}{\partial x_n} = 0 \text{ for } x_n \le x_n^{\,c}$$

$$\frac{\partial q}{\partial z} < 0, \frac{\partial^2 q}{\partial z^2} < 0 \text{ for } z < z^c; \ \frac{\partial q}{\partial z} = 0 \text{ for } z \ge z^c$$

$$\frac{\partial q}{\partial t_n^h} < 0, \frac{\partial^2 q}{\partial t_n^{h2}} < 0 \text{ for } t_n^h < t_n^{h\,c}; \ \frac{\partial q}{\partial t_n^h} = 0 \text{ for } t_m^h \ge t_m^{h\,c}$$

$$\frac{\partial q}{\partial t_k} > 0, \frac{\partial^2 q}{\partial t_k^{\,2}} > 0 \text{ for } t_k > t_k^{\,c}; \ \frac{\partial q}{\partial t_k} = 0 \text{ for } t_k \le t_k^{\,c}$$

The general and good-specific ω parameters serve to increase the observability of any given allocation and thus have the same sign as the marginal probability of detection for each good. The second derivatives indicate that an increase in $x_n^{\,c}$, z^c or $t_m^{h\,c}$ or a decrease in $t_k^{\,c}$ will increase the marginal probability of detection; this is because any given non-cooperative action is now further from the cooperative allocation.

I assume no costly monitoring technologies are available, *i.e.* the **ω** factors are characteristic of the specific goods in question, the marriage match and/or the specific migration opportunity and cannot be affected by either player. This assumption should not alter the general theoretical results, provided any available monitoring technologies are either prohibitively costly or cannot fully reveal all hidden actions.

Because changes in t_n^w are exactly proportional to changes in x_n, I assume that the choice of t_n^w does not have an independent effect on the probability of detection, *i.e.* changes in t_n^w do not affect q, holding x_n constant. It should be noted that, although the husband would also agree to an increase in x_n via an increase in t_n^w, he would only agree to this arrangement if all other allocations were held constant. Thus, in the non-cooperative case, an increase in x_n is indicative of a decrease in t_n^h and, consequently, the probability of detection must be increasing in x_n. The probability of detection is also increasing in t_k because, for player n, the individual utilitymaximizing value is greater than the cooperative value. Conversely, the optimal values of z and t_n^h are less than the cooperative values, and thus any increase in z or t_n^h will decrease the probability of detection. For simplicity, I have assumed that the probability of detection is zero for any value of z or t_n^h greater than z^c or $t_n^{h\,c}$, respectively, and any value of x_n or t_k less than x_n^c or t_k^c, respectively, because any allocations satisfying these conditions would increase the utility of player m. In practice, this assumption simply assures that the wife would not be punished for any non-cooperative behavior that benefits her spouse.

Definition. $\{s^{**}, t_m^{w\,**}, t_n^{w\,**}, t_n^{h\,**}, t_k^{**}\}$ are the Pareto optimal cooperative allocations that would be obtained in the absence of imperfect information, *i.e.* the allocations consistent with [2] in Section 3. Ψ^{**} is the surplus obtained in the absence of imperfect information.

Proposition 3. When player n plays *don't cooperate* and player m plays *cooperate* (and *punish*) in equilibrium, the allocations are not first-best Pareto efficient; that is, $\Psi < \Psi^{**}$. Furthermore, Ψ increases as the difference between the cooperative allocations and player n's non-cooperative allocations decreases.

Proof. In a (*cooperate*, *cooperate*) equilibrium, maximization of expected future payoffs yields the same allocations as maximization of instantaneous utility in any given period. However, in a (*cooperate*, *don't cooperate*) equilibrium, there is uncertainty about what payoffs will be realized in subsequent periods. Thus, when players maximize expected utility, the desire to smooth utility across periods creates a wedge between the optimal actions in the current period and the optimal actions over the infinite future. This implies that individual utility is not maximized in any one period for either player, and it must be the case that $\Psi < \Psi^{**}$. When player n's non-cooperative allocations are closer to the cooperative allocations, q is lower in equilibrium. The value of smoothing utility across periods is therefore lower for both players, and allocations will be closer to the unconstrained Pareto optimum in each period, *i.e.* Ψ will be closer to Ψ^{**}. ■

In contrast to de Laat (2005), the welfare loss in this case stems not from investment in costly monitoring technologies but from the fact that, given a sufficiently low probability of detection, the migrant cannot induce his wife to behave cooperatively.

Proposition 4. When player n plays *don't cooperate* and player m plays *cooperate* (and *punish*) in equilibrium, it must be the case that $s^c < s^{**}$.

Proof by Contradiction. Suppose that $s^c = s^{**}$. Consider an alternative transfer, $s^a < s^{**}$. Provided that "don't cooperate" is still an equilibrium strategy, the values of t_n^h and t_n^w, t_k chosen by player n under this alternative must be higher and lower, respectively, than the values chosen under $s^c = s^{**}$ (see Technical Appendix for derivation). For this reason, $s^a < s^{**}$ also yields a higher value of Ψ than $s^c = s^{**}$, which implies that both players can achieve a weakly greater payoff under this alternative set of cooperative allocations. But then $s^c > s^{**}$ cannot be constrained Pareto optimal. ■

The optimal values of $\{z^c, t_n^{w\,c}, t_n^{h\,c}, t_k^c\}$ relative to $\{z^{**}, t_n^{w\,**}, t_n^{h\,**}, t_k^{**}\}$ depend on the magnitude of player n's response to changes in the cooperative allocations. If $dy^*/dy^c > 1$ ($dy^*/dy^c < 1$), then $y^c > y^{**}$ ($y^c < y^{**}$) for $y \in \{z, t_n^w, t_n^h, t_k\}$. That is, if increasing $t_n^{h\,c}$ or z^c (decreasing $t_n^{w\,c}$ or t_k^c) brings the non-cooperative allocations closer to the cooperative allocations, then the optimal $t_n^{h\,c}$ and z^c ($t_n^{w\,c}$ and t_k^c) will be higher (lower) than the values that would be obtained in the absence of imperfect information. Alternatively, if decreasing $t_n^{h\,c}$ or z^c (increasing $t_n^{w\,c}$ or t_k^c) brings the non-cooperative allocations closer to the cooperative allocations, then the optimal $t_n^{h\,c}$ and z^c ($t_n^{w\,c}$ and t_k^c) will be lower (higher) than the values that would be obtained in the absence of imperfect information. In the latter case, the migrant is willing to pay a premium (decreasing $t_n^{h\,c}$, z^c and increasing $t_n^{w\,c}$, t_k^c reduces the migrant's utility) to mitigate moral hazard on the part of his spouse and thereby reduce uncertainty in his continuation payoff.

Proposition 5. When player *n* plays "don't cooperate" and player *m* plays "cooperate" in equilibrium, it must be the case that $t_n^{w\,*} \geq t_n^{w\,**}$, $z \leq z^{**}$, $t_n^{h\,*} \leq t_n^{h\,**}$, and $t_k^{*} \geq t_k^{**}$.

Proof. When $\{t_n^{w\,c} \geq t_n^{w\,**}, z^c \leq z^{**}, t_n^{h\,c} \leq t_n^{h\,**}, t_k^c \geq t_k^{**}\}$, this follows directly from Proposition 2. A proof by contradiction is used for the case of $\{t_n^{w\,c} < t_n^{w\,**}, z^c > z^{**}, t_n^{h\,c} > t_n^{h\,**}, t_k^c < t_k^{**}\}$. Suppose that, by threatening the maximum credible punishment and playing $\{s^c \leq s^{**}, t_n^{w\,c} < t_n^{w\,**}, z^c > z^{**}, t_n^{h\,c} > t_n^{h\,**}, t_k^c < t_k^{**}\}$, the migrant could induce his spouse to choose $\{t_n^{w\,c} < t_n^{w\,*} < t_n^{w\,**}, z^c > z^{*} > z^{**}, t_n^{h\,c} > t_n^{h\,*} > t_n^{h\,**}, t_k^c < t_k^{*} < t_k^{**}\}$. Then, in the presence of imperfect information, the migrant achieves strictly higher utility, and his spouse achieves strictly lower utility. But then $\{s^{**}, t_n^{w\,**}, z^{**}, t_n^{h\,**}, t_k^{**}\}$ could not be the allocations that would have been obtained in the absence of imperfect information. That is, if player *n* is willing to play $\{t_n^{w\,c} < t_n^{w\,*} < t_n^{w\,**}, z^c > z^{*} > z^{**}, t_n^{h\,c} > t_n^{h\,*} > t_n^{h\,**}, t_k^c < t_k^{*} < t_k^{**}\}$ when monitoring is imperfect, these same allocations should have also been feasible under perfect information. ■

Finally, note that, for allocations that are very easily monitored, *i.e.* actions with a high marginal probability of detection, the non-cooperative value will be identical to the cooperative value. However, at least one of the conditions, $t_n^{w\,*} \geq t_n^{w\,**}$, $z \leq z^{**}$, $t_n^{h\,*} \leq t_n^{h\,**}$, and $t_k^{*} \geq t_k^{**}$, must be a strict inequality in order for *don't cooperate* to yield a higher payoff than *cooperate* for player *n*.

Testable Implications

The equilibrium strategies discussed above are consistent with the empirical results presented in the previous section. Non-migrant spouses reduce own household labor and increase child household labor. Allocations that are easily observed – child schooling and health – exhibit no change with migration, conditional on income, but allocations that are difficult to verify – participation in household chores – exhibit relatively large changes, even on the extensive margin. I now examine how player *n*'s optimal actions vary with the parameters of the model. Player *n*'s Cournot reaction function is determined as follows (for simplicity, derivations are for the case where punishment lasts only one period, $f = 1$):

$$\max\ V_n = U_n(x_n, z, t_n, t_k) + [\delta/(1-\delta)][(1-q)U_n(x_n, z, t_n, t_k) + qU_n(x_n', z, t_n, t_k)],$$

where $q = q(x_n, z, t_n^h, t_k; x_n^{\ c}, z^c, t_n^{hc}, t_k^{\ c}, \omega_q, \omega_x, \omega_z, \omega_h, \omega_k)$,

$$x_n = w_n t_n^w + s^c,\ x_n' = w_n t_n^w + s',\ t_n = t_n^h + \mu_n t_n^w \text{ and } z = z(t_n^h, t_k; \tau_n, \tau_k)$$

Transfers from the husband, s^c and s', are taken as given. The first order conditions are

$$\left(\frac{\partial U_n}{\partial t_n} + \frac{\partial U_n}{\partial x_n} w_n\right) - \delta q\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) - \frac{\partial q}{\partial x_n} w_n \delta(U_n - U_n^{'}) = 0,$$

$$\left(\frac{\partial U_n}{\partial t_n}\mu_n + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right) - \left(\frac{\partial q}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_n^h}\right)\delta(U_n - U_n^{'}) = 0 \text{ and}$$

$$\left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right) - \left(\frac{\partial q}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)\delta(U_n - U_n^{'}) = 0.$$

Clearly, the optimal amount of household labor provided by the non-migrant spouse exceeds the amount she would provide when $q = 0$ because the gains from decreasing t_n^h are offset by an increase in the probability of detection. Whether the optimal amount of child household labor is higher or lower than when $q = 0$ depends on both the productivity of the child and the probability of detection for t_k relative to z. The optimal amount of wage labor will be less than that chosen when $q = 0$ only if the expected punishment for increasing t_n^w exceeds the gain to smoothing consumption between states. I assume that the parameters are such that the values of t_k and t_n^w are less than the values that would be chosen when $q = 0$.

Comparative statics (see Technical Appendix for derivation and full set of assumptions) are utilized to derive the following testable implications:

$$\frac{\partial t_n^w}{\partial \omega_q} < 0, \ \frac{\partial t_n^h}{\partial \omega_q} > 0 \text{ and } \frac{\partial t_k}{\partial \omega_q} < 0$$
$$\frac{\partial t_n^w}{\partial \omega_x} < 0, \ \frac{\partial t_n^h}{\partial \omega_x} > 0 \text{ and } \frac{\partial t_k}{\partial \omega_x} < 0$$
$$\frac{\partial t_n^w}{\partial \omega_\mu} > 0, \ \frac{\partial t_n^h}{\partial \omega_\mu} < 0 \text{ and } \frac{\partial t_k}{\partial \omega_\mu} > 0$$
$$\frac{\partial t_n^w}{\partial \tau_n} < 0, \ \frac{\partial t_n^h}{\partial \tau_n} > 0 \text{ and } \frac{\partial t_k}{\partial \tau_n} < 0$$
$$\frac{\partial t_n^w}{\partial \tau_k} > 0, \ \frac{\partial t_n^h}{\partial \tau_k} < 0 \text{ and } \frac{\partial t_k}{\partial \tau_k} > 0$$

Variation in the general factor ω_q can be thought of as differences across households in the distance and duration of migration episodes. More frequent return visits allow more frequent observations of intrahousehold allocations and increase both the overall and marginal probability of detecting any given deviation. An increase in ω_q brings all allocations closer to the cooperative values.

Private consumption is often in the form of durable items (*e.g.* clothing) which can be easily observed. To examine the case where deviations in x_n are easily detected relative to other allocations, we can consider the effect of an increase in ω_x. Under the assumption that the expected punishment for increasing t_n^w exceeds the gain to smoothing consumption between states, an increase in the probability of detection specifically for private consumption reduces the optimal value of t_n^w and thus x_n, bringing both closer to their cooperative values. The reduction in t_n^w reduces the level of disutility associated with labor hours and thus makes an increase in t_n^h in less costly which, in turn, makes a decrease in t_k less costly as well. An increase in ω_x thus brings all allocations closer to the cooperative values. The magnitude of these effects, however, is smaller than the effect of an increase in ω_q; this is because a general increase in the probability of detection induces more feedback effects between allocations.

In contrast, an increase in ω_z increases z but has an ambiguous effect on t_n^h and t_k. Holding constant the marginal probability of detection for time inputs, increasing the observability of z may actually induce additional reallocation of household labor between mothers and children. The marginal productivity of mothers and children in the household will also affect non-cooperative behavior via changes in the expected utility gain for any given deviation. When mothers are more productive, a decrease in t_n^h results in a larger reduction in z and thus a larger increase in the probability of detection; however, any given level of z can be provided with less labor and less disutility. The converse is true for children. Accordingly, the formal comparative statics indicate that an increase in mothers' productivity brings labor allocations closer to the cooperative values, whereas an increase in children's productivity does the opposite. Finally, an increase in mothers' relative disutility of own household labor pulls labor allocations further away from the cooperative values. Generalizing to multiple household goods and multiple children, this suggests that deviations in mothers' and children's labor inputs should be larger for goods in which the child has relatively higher productivity and for goods which involve higher disutility for own time spent in production.

VI. Tests of the Non-Cooperative Model

Non-cooperative behavior on the part of the mother implies a reduction in her own household labor and an increase in children's household labor. This is consistent with the findings in Tables 4 and 5. However, the focus on a composite measure of household chores masks substantial variation in time allocation, as indicated in Table 8. One implication of the non-cooperative model is that children who are more productive will be more likely to increase their time in household production, and this effect will be larger for those goods for which children are relatively more productive. As a proxy for productivity, we can examine the frequency with which children engage in various tasks. Laundry is the most common household chore, followed by food preparation. This ordering does not vary by gender of the child, but girls are more likely to be engaged in all three household chores.

A second implication of the non-cooperative model is that changes in household labor will be largest for those tasks which provide the highest disutility. One measure of the degree of disutility associated with a task is the extent to which it depletes body mass. To determine the effort expended for each of the three household chores, I estimate a health production function. Pitt *et. al.* (1990), however, find that work activities and calories are allocated among household members according to unmeasured health-related endowments. Therefore, following Foster and Rosenzweig (1994), variables reflecting household budget constraints – productive assets, household composition, food prices and month and year of survey – are utilized as instruments for activities, calorie intake, and lagged health. Table 7 presents two-stage least squares estimates of the health (BMI) production function. Because data on actual hours in household chores were collected inconsistently, these activities are included as binary regressors, and thus the effect of household chores on BMI can only be estimated imprecisely. Nonetheless, laundry appears to be the most energy intensive household chore.

The child fixed-effects estimates in Table 8 are consistent with these predictions. Longer migration episodes significantly increase the probability that daughters do laundry and have the opposite and also statistically significant effect for sons. The point estimates are quite large in magnitude – the average marginal effects indicate that the probability that sons do laundry is 6.1 percentage points lower and the probability that daughters do laundry is 19.1 percentage points higher, compared to the baseline in which approximately 7.3% of boys and 18.7% of girls aged 6-16 do laundry. The scope for non-cooperative behavior also increases with the number of children in the household because the probability of detection depends on the magnitude of each deviation. However, while the number of children will increase the probability and frequency of non-cooperative behavior, the magnitude of any single deviation is likely to be smaller when there are more children in the household. Thus, for changes on extensive margins, *i.e.* the probability of engaging in some household task, the number of siblings should reinforce non-cooperative behavior, whereas the opposite would be true for changes on intensive margins such as nutritional intake. Consistent with this, results in Table 8 suggest that siblings reinforce the effect of months away, with own gender siblings having a larger effect, although the point estimates are not statistically significant.

The findings for laundry are mirrored in the estimates for food preparation; girls are more likely to be engaged in this task when fathers are away, and the opposite is true for boys. The point estimates are generally smaller than those for laundry, which is consistent with descriptive evidence that children have lower productivity in food preparation than in laundry. However, it also appears that simultaneous changes in all three household chores may be somewhat offsetting. The average marginal effect of paternal migration is much larger for each of the specific chores than for the composite measure of chores presented in Table 4. Furthermore, the probability that boys purchase food for the household increases when fathers are away, and the opposite is true for girls. This follows the gendered division of household labor among adults; the most common chore reported by fathers is purchasing food, and laundry is the least common, with the opposite being true for mothers. Turning to mothers' detailed time allocation, Table 9 shows that mothers are less likely to prepare food or do laundry when fathers migrate, and this effect is increasing in the number of months that the father is away. As with the findings for daughters, the average marginal effects are larger for laundry than for food preparation. The probability that mothers purchase food is also decreasing in months away, but this effect does not dominate the direct positive effect for any value of months away in the relevant range (1-12).

When a specific allocation can be more easily monitored, the non-cooperative value chosen by the mother will be closer to the cooperative value negotiated prior to migration. This is also true for expenditures/allocations that can be mapped directly into observable outcomes – in this case, school attendance. Health (body mass index) can also be easily observed but, with a stochastic production function, it may be more difficult to detect whether changes are due to non-cooperative behavior or unobservable shocks and/or endowments. Indeed, the average marginal effects of migration on health are slightly larger, in percentage terms, than those for school enrollment (see Table 4), although both are small in magnitude and not statistically significant. However, time spent in productive activities also affects an individual's health. To the extent that health outcomes can be monitored, inputs to the health production function, *i.e.* labor hours and nutrition, must be adjusted simultaneously in order to keep observable health measures within a certain range. Estimates presented in Table 10 depict exactly this. Migration of the father has large and statistically significant effects on children's nutritional intake. The direct effect is positive for boys and negative for girls, but age effects work in the opposite direction such that the marginal effects are positive for girls and negative for boys at all ages. Estimated coefficients are larger in magnitude for girls than for boys, consistent with the finding that, in absolute terms, changes in household labor are larger for girls than for boys. Own-gender sibling effects are opposite in sign to age effects, suggesting that the magnitude of deviations declines with the number of children in the household, as predicted. Months away does not have a statistically significant effect on nutrition, which suggests that the intensity of household activities varies predominantly with age, although children of all ages are equally likely to shift on the extensive margin.

Robustness Checks

Next, I examine the possibility that the results are driven by unobserved changes in income or bargaining power rather than by non-cooperative behavior. Changes in the distribution of household bargaining power would not be captured by person-fixed effects, and the variables utilized to control for changes in wages may have significant measurement error. By utilizing a sample of migrants who were home for the entire week preceding the survey, I can largely eliminate the scope for non-cooperative behavior and replace fathers' time in household production. If the remaining factors, change in income and change in bargaining power, are the main causes of the changes in time allocation estimated above, this sample should yield similar results. Results presented in Table 11 indicate no significant effects of migration on either mothers' or children's household labor when migrant fathers are present. The point estimates are much smaller in magnitude and, in fact, tend be opposite in sign (estimates for children's participation in specific household chores are again similar and not presented here). These findings support the conclusion of non-cooperative behavior.

Finally, to determine the generality of the main results, I further restrict the sample of migrant households to those in which the father migrates in multiple survey periods. If migration is less likely to occur in households with strong tendencies towards non-cooperation, households in the restricted sample should exhibit a lesser degree of non-cooperative behavior and therefore smaller changes in time allocation. In fact, estimates in Table 12 suggest the opposite. The negative effect of months away on mothers' household labor is more pronounced, and the average marginal effect is much larger and statistically significant. Estimates for children are less precise than for the main sample in Table 4 but display the same sign pattern, and the average marginal effects are much larger. Taken together, these findings suggest that repeat migration in fact increases the scope for non-cooperative.

VII. Conclusion

Non-cooperative behavior among spouses is common in anecdotes but difficult to identify in typical survey data. In this paper, I use the incidence of migration to examine such behavior. Migration by one spouse presents a clear opportunity for non-cooperation by introducing imperfect monitoring and increasing the transaction costs associated with enforcing a cooperative equilibrium. I find evidence that wives of migrants do attempt to conceal allocations from their husbands. In particular, mothers shift household chores to children and consume more leisure themselves when fathers are away. To limit the probability that such behavior is detected, mothers also adjust the distribution of nutrition among children in order to keep observable health outcomes stable. This change in household labor is not consistent with a simple reallocation of time in order to compensate for the father's absence, nor is it consistent with a pure income effect. Furthermore, given existing evidence on rural Chinese households (Qian, 2005), the observed stability in children's health and schooling outcomes is not consistent with an increase in mother's bargaining power due to the absence of the father. These conclusions are also robust to several alternative interpretations: (1) an increase in the demand for mothers' time in non-enumerated household tasks, (2) unobserved changes in bargaining power and/or inadequate controls for changes in income, and (3) self-selection of migrants on the propensity for non-cooperative behavior.

The type of non-cooperative behavior observed in this setting appears relatively innocuous; children's school enrollment is unaffected by the changes in time allocation, and changes in household labor are compensated by changes in nutritional intake in order to maintain child health. However, increasing opportunities for international migration, *i.e.* migration over longer distances and for longer periods of time, will exacerbate informational asymmetries. The ultimate effect on intrahousehold allocation will depend on the capacity for monitoring and the preferences of decision-makers remaining in the sending household. To the extent that this information problem constrains the allocation of remittance income to easily observable goods, non-cooperative behavior may generate inefficiencies in investment and hinder growth (see Chami *et. al.*, 2003). Development agencies may also wish to consider how the efficacy of targeted transfers and subsidies is affected by the transparency of those income sources.

Further research should consider the effect of non-cooperative behavior on a broader range of allocations which have larger implications for economic growth, *e.g.* schooling-related expenditures and investments in income-generating activities. To do so, it will be crucial to understand how remittance flows are affected by non-cooperative behavior on the part of both recipients and senders. If migrants' earnings are difficult for sending households to monitor, migrants face a trade-off when determining the value of remittance flows. An increase in remittances will increase the migrant's bargaining power in the household but, because the migrant must then bargain with other household members over the allocation of this income, remittance flows will effectively be taxed, even when there is no non-cooperative behavior on the part of recipients. Better data on the distance of migration and frequency of visits would shed light on the sensitivity of non-cooperative behavior to the capacity for monitoring and permit clearer extrapolation to households in which all decision-makers are co-resident.

REFERENCES

Ashraf, Nava. 2004. "Spousal Control and Intra-household Decision Making: An Experimental Study in the Philippines." Mimeo, Harvard University.

Brownlee, Patrick and Colleen Mitchell. 1997. "Migration Issues in the Asia Pacific." APMRN Secretariat Centre for Multicultural Studies in conjunction with UNESCO.

Chami, Ralph, Connel Fullenkamp and Samir Jahjah. 2003. "Are Immigrant Remittance Flows a Source of Capital for Development?" International Monetary Fund Working Paper 03/189.

Chen, Joyce. 2005. "Dads, Disease and Death: Decomposing Daughter Discrimination." *CID Graduate Student and Postdoctoral Fellow Working Paper Series*, No. 8.

de Brauw, Alan and John Giles. 2005. "Migrant Opportunity and the Educational Attainment of Youth in Rural China." Mimeo, Michigan State University.

de Laat, Joost. 2005. "Moral Hazard and Costly Monitoring: The Case of Split Migrants in Kenya." Mimeo, Brown University.

Dubois, Pierre and Ethan Ligon. 2004. "Incentives and Nutrition for Rotten Kids: Intrahousehold Food Allocation in the Philippines." Mimeo, University of California, Berkeley.

Duflo, Esther. 2003. "Grandmothers and Granddaughters: Old Age Pension and Intra-Household Allocation in South Africa." *World Bank Economic Review*. 17(1), 1-25.

Edwards, Alejandra Cox and Manuelita Ureta. 2003. "International Migration, Remittances and Schooling: Evidence from El Salvador." *Journal of Development Economics*. 72(2), 429-461.

Foster, Andrew and Mark Rosenzweig. 1994. "A Test for Moral Hazard in the Labor Market: Contractual Arrangements, Effort, and Health." *The Review of Economics and Statistics*. 76(2), 213-227.

Liang, Zai and Zhongdong Ma. 2004. "China's Floating Population: New Evidence from the 2000 Census." *Population and Development Review*. 26(1), 1-29.

Lundberg, Shelly and Robert Pollak. 1993. "Separate Spheres Bargaining and the Marriage Market." *Journal of Political Economy*. 101(6), 988-1010.

Manser, Marilyn and Murray Brown. 1980. "Marriage and Household Decision-Making: A Bargaining Analysis." *International Economic Review*. 21(1), 31-44.

McElroy, Marjorie and Mary Jean Horney. 1981. "Nash-Bargained Household Decisions: Toward a Generalization of the Theory of Demand." *International Economic Review*. 22(2), 333-349.

Pitt, Mark, Mark Rosenzweig and Nazmul Hassan. 1990. "Productivity, Health and Inequality in the Intrahousehold Distribution of Food in Low-income Countries." *American Economic Review*. 80(5), 1139-1156.

Qian, Nancy. 2005. "Missing Women and the Price of Tea in China: The Effect of Relative Female Income on Sex Imbalance." Mimeo, Massachusetts Institute of Technology.

Sukamdi, Abdul Harris and Patrick Brownlee. 1998. "Labour Migration in Indonesia: Policies and Practices." Population Studies Center Gadjah Mada University in conjunction with the UNESCO.

Thomas, Duncan. 1990. "Intrahousehold Resource Allocation: An Inferential Approach." *Journal of Human Resources*. 25(4), 635-664.

The United Nations. *International Migration Report 2002*. Department of Economic and Social Affairs.

Yang, Dean. 2004. "International Migration, Human Capital and Entrepreneurship: Evidence from Philippine Migrants' Exchange Rate Shocks." Mimeo, University of Michigan.

Table 1. Characteristics of Children Age 6-16 by Gender and Migrant Status

	Father Never Migrates		Father Migrates at Least Once		Father Currently Away	
	Sons	Daughters	Sons	Daughters	Sons	Daughters
Age	11.28	11.40	11.46 *	11.34	11.63	11.50
	(3.093)	(3.074)	(3.073)	(3.062)	(3.131)	(3.121)
School Enrollment	0.861	0.827	0.882 *	0.868 ***	0.893	0.884 **
	(0.346)	(0.378)	(0.323)	(0.339)	(0.310)	(0.321)
Body Mass Index	17.14	17.11	17.02	17.21	16.88	17.33
	(3.609)	(2.937)	(2.527)	(3.124)	(2.588)	(2.964)
Upper Arm Circumference	19.04	19.03	18.87	19.11	18.97	19.06
	(4.423)	(3.628)	(3.409)	(3.517)	(3.521)	(3.926)
Skin Fold	8.320	10.29	8.573	10.58	8.952	11.26 *
	(4.906)	(5.622)	(4.951)	(5.276)	(5.104)	(5.366)
Buy Food for the Hh	0.024	0.032	0.021	0.040	0.021	0.041
	(0.153)	(0.176)	(0.142)	(0.197)	(0.143)	(0.198)
Prepare Food for the Hh	0.057	0.117	0.059	0.106	0.087	0.130
	(0.231)	(0.321)	(0.236)	(0.308)	(0.283)	(0.338)
Do Laundry for the Hh	0.073	0.187	0.061	0.159 *	0.084	0.173
	(0.259)	(0.390)	(0.239)	(0.366)	(0.278)	(0.379)
Do Any Chores	0.115	0.228	0.102	0.201 *	0.125	0.209
(buy/prep food or laundry)	(0.319)	(0.420)	(0.303)	(0.401)	(0.332)	(0.408)
Engage in Other Work	0.063	0.077	0.050	0.062 *	0.043	0.068
	(0.243)	(0.267)	(0.219)	(0.241)	(0.203)	(0.252)
Daily Calorie Intake	1851	1711	1880	1724	1838	1663
	(636.3)	(551.2)	(595.1)	(532.2)	(551.3)	(706.9)
Daily Protein Intake	63.01	58.14	63.84	57.95	64.49	54.68 *
	(25.40)	(21.79)	(22.82)	(19.59)	(23.11)	(21.63)
Daily Fat Intake	28.37	25.99	29.20	25.95	35.01 ***	22.81 *
	(26.71)	(22.24)	(24.34)	(20.63)	(27.89)	(17.81)
Months Away in the Year					6.681	6.340
					(3.908)	(3.923)
Number of Observations	4474	4116	1073	969	210	162

Notes: Standard deviations reported in parentheses. (*) indicates significantly different from column [1] or column [2] at the 10%, (**) 5% or (***) 1% level. Observations at the person-year level.

Table 2. Characteristics of Households by Migrant Status

	Husband Never Migrates	Husband Migrates at Least Once	Husband Currently Away
Number of Children	2.116	2.099	2.000 **
	(0.941)	(0.927)	(0.923)
Sex Ratio of Children	0.544	0.537	0.577
	(0.357)	(0.354)	(0.361)
% with Only One Child	0.278	0.278	0.318
	(0.448)	(0.448)	(0.467)
Mother's Age	38.34	38.07	38.20
	(6.374)	(5.743)	(6.075)
Father's Age	40.26	39.80 **	39.95
	(7.018)	(6.078)	(6.595)
Mother's Schooling	5.636	6.285 ***	6.195 **
	(4.129)	(4.061)	(3.744)
Father's Schooling	7.539	7.989 ***	7.899 *
	(3.497)	(3.232)	(3.034)
Household Size	4.313	4.237 **	4.106 ***
	(1.118)	(1.027)	(0.969)
Mother's Wage	9.072	9.060	9.154
	(12.79)	(13.30)	(9.439)
Father's Wage	11.79	12.68 *	15.24 ***
	(21.42)	(16.56)	(15.55)
Area of Owned Home	66.31	64.85	65.12
	(54.86)	(57.73)	(50.82)
Farm Land	3.636	2.810 ***	3.121
	(8.807)	(4.697)	(6.662)
Value of Business Equip.	213.1	209.3	58.77 ***
	(2234)	(3087)	(388.4)
Adj. Per Capita Hh Income	1344	1430 ***	1590 ***
	(1053)	(1045)	(1052)
Months Away in the Year			6.606
			(3.878)
Number of Observations	5666	1344	264

Notes: Standard deviations reported in parentheses. (*) indicates significantly different from column [1] at the 10%, (**) 5% or (***) 1% level. Observations at the household-year level.

Table 3. Mothers' Outcomes of Interest by Migrant Status

	Husband Does Not Migrate	Husband Migrates at Least Once	Husband Currently Away
Total Work Hours	43.50	44.78	44.43
(excl. household chores)	(29.25)	(30.38)	(29.48)
Body Mass Index	22.41	22.27	22.31
	(2.963)	(2.819)	(2.841)
Upper Arm Circumference	25.08	25.01	25.14 **
	(3.090)	(2.767)	(2.729)
Skin Fold	14.67	14.78	15.78
	(7.086)	(6.989)	(7.094)
Daily Calorie Intake	2119	2121	2027 **
	(660.8)	(598.7)	(633.8)
Daily Protein Intake	71.62	71.40	69.40
	(26.13)	(21.89)	(24.10)
Daily Fat Intake	29.99	31.36 *	31.72
	(26.38)	(24.06)	(26.67)
Buy Food for the Hh	0.614	0.673 ***	0.760 ***
	(0.487)	(0.469)	(0.428)
Prepare Food for the Hh	0.912	0.916	0.939 *
	(0.284)	(0.277)	(0.240)
Do Laundry for the Hh	0.915	0.929 *	0.935
	(0.279)	(0.257)	(0.247)
Do Any Chores	0.974	0.969	0.973
(buy/prep food or laundry)	(0.158)	(0.172)	(0.162)
Number of Observations	5677	1344	264

Notes: Standard deviations reported in parentheses. (*) indicates significantly different from column [1] at the 10%, (**) 5% or (***) 1% level. Observations at the person-year level.

Table 4. Outcomes for Children Age 6-16, Child-Fixed Effects Estimates

	I Do Any Chores	II Engage in Other Work	III School Enrollment	IV Body Mass Index
Father Away	0.255	0.032	0.056	-0.671
	(0.209)	(0.119)	(0.172)	(0.844)
Months Father Away	-0.043	0.000	-0.053	0.203
	(0.053)	(0.026)	(0.045)	(0.253)
Months Away Squared	0.003	-0.001	0.005 *	-0.025
	(0.004)	(0.002)	(0.003)	(0.022)
(Age-6)*Away	-0.006	0.029	0.019	0.155
	(0.049)	(0.029)	(0.045)	(0.257)
(Age-6) Squared*Away	0.000	-0.003	0.000	-0.015
	(0.004)	(0.003)	(0.004)	(0.026)
Male Siblings*Away	-0.056	-0.034	0.016	0.525
	(0.079)	(0.031)	(0.068)	(0.362)
Female Siblings*Away	-0.052	-0.060	-0.048	0.388
	(0.047)	(0.052)	(0.078)	(0.326)
Marginal Effect of Away	**0.008**	**0.004**	**0.006**	**0.351**
	(0.080)	**(0.041)**	**(0.071)**	**(0.366)**
Relative Effect for Girls				
Father Away	-0.415	0.060	-0.041	0.657
	(0.299)	(0.161)	(0.275)	(1.750)
Months Father Away	0.100	-0.025	0.008	-0.180
	(0.077)	(0.047)	(0.065)	(0.447)
Months Away Squared	-0.007	0.002	-0.001	0.020
	(0.006)	(0.003)	(0.005)	(0.038)
(Age-6)*Away	-0.019	0.023	-0.030	-0.539
	(0.077)	(0.037)	(0.073)	(0.522)
(Age-6) Squared*Away	0.001	-0.003	0.004	0.063
	(0.007)	(0.004)	(0.006)	(0.052)
Male Siblings*Away	0.137	0.066	-0.074	-0.490
	(0.111)	(0.052)	(0.091)	(0.950)
Female Siblings*Away	0.101	-0.050	0.051	-0.147
	(0.109)	(0.068)	(0.105)	(0.754)
Marginal Effect of Away	**-0.005**	**0.017**	**-0.068**	**-0.898**
	(0.135)	**(0.062)**	**(0.116)**	**(0.852)**
Number of Observations	8739	9794	9056	6121

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for age of parents, assets owned, household size, parents' wages, month and year of survey, and community-year fixed effects.

Table 5. Mothers' Time Allocation, Mother-Fixed Effects Estimates

	I	II	III
	Do Any Chores	Work Hours (excl. chores)	Body Mass Index
Father Away	0.077 *	12.19	0.317
	(0.046)	(8.002)	(0.442)
Months Father Away	-0.031	-5.579 *	-0.071
	(0.019)	(3.085)	(0.172)
Months Away Squared	0.002 **	0.459 *	0.002
	(0.001)	(0.241)	(0.014)
Marginal Effect of Away	**-0.041**	**-4.395**	**-0.106**
	(0.030)	**(4.274)**	**(0.241)**
Number of Observations	6450	5996	5777

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects.

Table 6. Household Time Allocation, Father Debilitated

	Do Any Chores		
	I	II	
	Mothers	Sons	Girls (Relative)
Father Debilitated	0.016	0.246 *	-0.461 *
	(0.017)	(0.137)	(0.258)
Days Father Debilitated	0.001	0.002	-0.005
	(0.002)	(0.006)	(0.010)
Days Debilitated Squared	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
(Age-6)*Sick		-0.064	0.188 *
		(0.052)	(0.096)
(Age-6) Squared*Sick		0.004	-0.015 *
		(0.005)	(0.009)
Male Siblings*Sick		-0.026	0.116
		(0.064)	(0.113)
Female Siblings*Sick		0.002	-0.129
		(0.049)	(0.087)
Marginal Effect of Sick	**0.025**	**-0.011**	**0.104**
	(0.016)	**(0.050)**	**(0.089)**
Number of Observations	5396	7393	

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 7. 2SLS Estimates
of BMI Production Function

Lagged BMI	0.8330	***
	(0.0327)	
Hours in Wage Labor	0.0012	
Professional and Administrative	(0.0052)	
Hours in Wage Labor	-0.0041	
Skilled and Semi-Skilled	(0.0067)	
Hours in Wage Labor	-0.0270	
Farmers, Fishermen, etc.	(0.0344)	
Hours in Wage Labor	0.0061	
Unskilled	(0.0057)	
Hours in Wage Labor	-0.0046	
Service and Other Misc.	(0.0066)	
Hours in Gardening	-0.0139	***
	(0.0039)	
Hours in Farming	-0.0051	*
	(0.0030)	
Hours in Livestock Care	-0.0370	***
	(0.0105)	
Hours in Fishing	0.0557	
	(0.0932)	
Hours in Handicrafts	0.0029	
	(0.0050)	
Buy Food for the Hh	-0.1588	
	(0.2238)	
Prepare Food for the Hh	0.8183	***
	(0.2540)	
Do Laundry for the Hh	-0.7689	***
	(0.2579)	
Daily Calorie Intake * 10^{-3}	0.1527	
	(0.1266)	
Age	0.0077	
	(0.0224)	
Age Squared	-0.0001	
	(0.0003)	
Female	0.2149	
	(0.1715)	
Constant	3.5513	***
	(0.6720)	
Number of Observations	3464	

Notes: Standard errors clustered at the individual level and reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Activities, calorie intake, and lagged health instrumented with assets, household composition, food prices, community of residence, and year and month of survey. Includes individuals age 16 to 60.

Table 8. Children's Detailed Time Allocation, Child-Fixed Effects Estimates

	I	II	III
	Buy Food	Prepare Food	Do Laundry
Father Away	0.030	0.253	0.228
	(0.084)	(0.168)	(0.195)
Months Father Away	0.024	-0.047	-0.084 *
	(0.017)	(0.048)	(0.051)
Months Away Squared	-0.002 **	0.003	0.007 *
	(0.001)	(0.004)	(0.004)
(Age-6)*Away	-0.018	-0.044	0.013
	(0.021)	(0.038)	(0.042)
(Age-6) Squared*Away	0.001	0.004	-0.001
	(0.002)	(0.004)	(0.004)
Male Siblings*Away	-0.008	0.001	-0.057
	(0.018)	(0.052)	(0.084)
Female Siblings*Away	-0.027	-0.006	-0.055
	(0.023)	(0.035)	(0.046)
Marginal Effect of Away	**0.046**	**-0.064**	**-0.061**
	(0.031)	**(0.087)**	**(0.072)**
Relative Effect for Girls			
Father Away	-0.247	-0.116	-0.365
	(0.154)	(0.208)	(0.287)
Months Father Away	-0.008	0.065	0.164 **
	(0.041)	(0.064)	(0.078)
Months Away Squared	0.000	-0.003	-0.013 **
	(0.003)	(0.005)	(0.006)
(Age-6)*Away	0.010	0.006	-0.015
	(0.039)	(0.054)	(0.070)
(Age-6) Squared*Away	0.000	0.000	0.002
	(0.003)	(0.005)	(0.006)
Male Siblings*Away	0.106 *	-0.091	0.034
	(0.056)	(0.078)	(0.117)
Female Siblings*Away	0.082	-0.077	0.142
	(0.063)	(0.079)	(0.112)
Marginal Effect of Away	**-0.136 ***	**0.120**	**0.191**
	(0.073)	**(0.112)**	**(0.123)**
Number of Observations	8723	8476	8329

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for age of parents, assets owned, household size, parents' wages, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 9. Mothers' Detailed Time Allocation, Mother-Fixed Effects Estimates

	I	II	III
	Buy Food	Prepare Food	Do Laundry
Father Away	0.125	0.105 *	0.067
	(0.147)	(0.062)	(0.056)
Months Father Away	-0.033	-0.045 *	-0.041
	(0.052)	(0.024)	(0.025)
Months Away Squared	0.003	0.003	0.003
	(0.004)	(0.002)	(0.002)
Marginal Effect of Away	**0.031**	**-0.059**	**-0.078 ***
	(0.070)	**(0.040)**	**(0.040)**
Number of Observations	6440	6430	6436

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects.

Table 10. Children's Nutrition, Child-Fixed Effects Estimates

	I Daily Calorie Intake	II Daily Protein Intake	III Daily Fat Intake
Father Away	288.0	18.51	7.558
	(276.6)	(13.94)	(11.34)
Months Father Away	10.96	-1.025	0.070
	(93.99)	(4.205)	(3.855)
Months Away Squared	-3.574	0.010	-0.117
	(8.319)	(0.355)	(0.327)
(Age-6)*Away	-181.0 **	-8.521 **	-3.800
	(88.37)	(3.841)	(3.372)
(Age-6) Squared*Away	18.11 **	0.867 **	0.387
	(9.166)	(0.418)	(0.360)
Male Siblings*Away	23.31	-6.843	-1.367
	(140.0)	(5.762)	(4.980)
Female Siblings*Away	187.3	5.489	6.205
	(132.6)	(5.389)	(4.782)
Marginal Effect of Away	**-139.7**	**-8.776**	**-4.167**
	(142.6)	**(5.501)**	**(4.937)**
Relative Effect for Girls			
Father Away	-409.8	-4.813	-12.96
	(444.8)	(20.32)	(15.33)
Months Father Away	53.31	0.160	3.424
	(138.5)	(5.914)	(4.959)
Months Away Squared	-2.433	-0.008	-0.113
	(10.98)	(0.457)	(0.396)
(Age-6)*Away	211.4 *	6.591	1.380
	(119.8)	(5.547)	(4.400)
(Age-6) Squared*Away	-24.68 **	-0.871	-0.104
	(12.38)	(0.561)	(0.462)
Male Siblings*Away	130.5	9.081	0.817
	(188.3)	(7.596)	(6.594)
Female Siblings*Away	-257.7	-5.289	-8.785
	(209.9)	(8.966)	(6.543)
Marginal Effect of Away	**160.5**	**5.985**	**6.033**
	(229.6)	**(9.167)**	**(6.901)**
Number of Observations	7303	7283	7173

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for age of parents, assets owned, household size, parents' wages, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 11. Household Time Allocation, Migrant Home at Survey

	Do Any Chores		
	I	II	
	Mothers	Sons	Girls (Relative)
Father Away	-0.008	0.147	0.035
	(0.047)	(0.132)	(0.207)
Months Father Away	0.008	0.017	-0.109
	(0.024)	(0.063)	(0.086)
Months Away Squared	-0.001	-0.003	0.011
	(0.002)	(0.005)	(0.007)
(Age-6)*Away		-0.050	0.015
		(0.044)	(0.072)
(Age-6) Squared*Away		0.004	-0.001
		(0.005)	(0.008)
Male Siblings*Away		-0.047	0.072
		(0.092)	(0.113)
Female Siblings*Away		-0.018	0.078
		(0.063)	(0.097)
Marginal Effect of Away	**0.020**	**-0.063**	**-0.049**
	(0.037)	**(0.111)**	**(0.148)**
Number of Observations	6405	8670	

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 12. Household Time Allocation, Father Migrates Multiple Times

	Do Any Chores		
	I	II	
	Mothers	Sons	Girls (Relative)
Father Away	0.113	-0.069	-0.314
	(0.075)	(0.290)	(0.435)
Months Father Away	-0.060 *	-0.005	0.110
	(0.032)	(0.079)	(0.117)
Months Away Squared	0.004 **	0.001	-0.008
	(0.002)	(0.006)	(0.009)
(Age-6)*Away		0.004	-0.009
		(0.070)	(0.099)
(Age-6) Squared*Away		-0.001	-0.002
		(0.006)	(0.009)
Male Siblings*Away		0.005	0.255
		(0.144)	(0.287)
Female Siblings*Away		0.024	-0.049
		(0.080)	(0.176)
Marginal Effect of Away	**-0.108 ****	**-0.040**	**0.056**
	(0.053)	**(0.132)**	**(0.220)**
Number of Observations	5604	7482	

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Technical Appendix

A. Cooperative Case – Unitary Household

First Order Conditions

$$\frac{\partial U}{\partial t_m} + \frac{\partial U}{\partial x_n} w_m = 0$$

$$\frac{\partial U}{\partial t_n} + \frac{\partial U}{\partial x_n} w_n = 0$$

$$\frac{\partial U}{\partial x_m} - \frac{\partial U}{\partial x_n} = 0$$

$$\frac{\partial U}{\partial t_m} \mu + \frac{\partial U}{\partial z} \frac{\partial z}{\partial t_m^h} = 0$$

$$\frac{\partial U}{\partial t_n} \mu + \frac{\partial U}{\partial z} \frac{\partial z}{\partial t_n^h} = 0$$

$$\frac{\partial U}{\partial t_k} + \frac{\partial U}{\partial z} \frac{\partial z}{\partial t_k} = 0$$

Assumptions: (1) All goods separable in utility.
(2) No complementarities in household production.

Let H denote the determinant of the Hessian, and define its elements as

$$h_{11} = \frac{\partial^2 U}{\partial t_m{}^2} + \frac{\partial^2 U}{\partial x_m{}^2} w_m{}^2 < 0$$

$$h_{12} = \frac{\partial^2 U}{\partial x_n{}^2} w_m w_n < 0$$

$$h_{13} = -\frac{\partial^2 U}{\partial x_n{}^2} w_m > 0$$

$$h_{14} = \frac{\partial^2 U}{\partial t_m{}^2} \mu < 0$$

$$h_{22} = \frac{\partial^2 U}{\partial t_n{}^2} + \frac{\partial^2 U}{\partial x_n{}^2} w_n{}^2 < 0$$

$$h_{23} = -\frac{\partial^2 U_n}{\partial x_n{}^2} w_n > 0$$

$$h_{25} = \frac{\partial^2 U}{\partial t_n{}^2} \mu < 0$$

$$h_{33} = \frac{\partial^2 U}{\partial {x_m}^2} + \frac{\partial^2 U}{\partial {x_n}^2} < 0$$

$$h_{44} = \frac{\partial^2 U}{\partial {t_m}^2}\mu^2 + \frac{\partial^2 U}{\partial z^2}\left(\frac{\partial z}{\partial t_m^h}\right)^2 + \frac{\partial U}{\partial z}\frac{\partial^2 z}{\partial {t_m^h}^2} < 0$$

$$h_{45} = \frac{\partial^2 U}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_n^h} < 0$$

$$h_{46} = \frac{\partial^2 U}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_k} < 0$$

$$h_{55} = \frac{\partial^2 U}{\partial {t_n}^2}\mu^2 + \frac{\partial^2 U}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial U}{\partial z}\frac{\partial^2 z}{\partial {t_n^h}^2} < 0$$

$$h_{56} = \frac{\partial^2 U}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k} < 0$$

$$h_{66} = \frac{\partial^2 U}{\partial {t_k}^2} + \frac{\partial^2 U}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial U}{\partial z}\frac{\partial^2 z}{\partial {t_k}^2} < 0$$

$$s_{15} = s_{16} = s_{24} = s_{26} = s_{34} = s_{35} = s_{36} = 0$$

$$\left.\frac{dt_m^h}{dw_m}\right|^{compensated} = \frac{1}{H}\ [((h25h33h56h46 - h25h33h66h45)h12 + (-h25h56h23h46 + h25h66h23h45)h13 + (h66h33h25^2 + h33h56^2h22 - h33h66h55h22 - h56^2h23^2 + h66h23^2h55)h14)a]$$

$$\left.\frac{dt_n^h}{dw_m}\right|^{compensated} = \frac{1}{H}\ [((-h25h33h46^2 + h25h33h66h44)h12 + (h25h23h46^2 - h25h23h66h44)h13 + (-h23^2h66h45 + h33h66h45h22 + h46h23^2h56 - h46h33h56h22)h14)a]$$

$$\left.\frac{dt_k}{dw_m}\right|^{compensated} = \frac{1}{H}\ [((h25h33h45h46 - h25h33h56h44)h12 + (-h25h23h45h46 + h25h23h56h44)h13 + (-h46h23^2h55 + h23^2h56h45 - h33h25^2h46 + h46h33h55h22 - h33h56h45h22)h14)a]$$

$\frac{dt_n^h}{dt_m^h} < 0$, $\frac{dt_k}{dt_m^h}$ *ambiguous*

where $a = -\frac{\partial U}{\partial x_n}$

$$\left.\frac{dt_n^h}{dw_m}\right|^{income}_{t_m^h} = \frac{1}{H}\ [((h12h25h33 - h13h25h23)h66b + (-h11h25h33 + h13^2h25)h66c + (h11h25h23 - h12h25h13)h66d]$$

$$\left.\frac{dt_k}{dw_m}\right|^{income}_{t_m^h} = \frac{1}{H}\ [((-h12h25h33 + h13h25h23)h56b + (h11h25h33 - h13^2h25)h56c + (-h11h25h23 + h12h25h13)h56d]$$

$\left.\frac{dt_n^h}{dw_m}\right|^{income}_{t_m^h} > 0$, $\left.\frac{dt_k}{dw_m}\right|^{income}_{t_m^h} < 0$

where $b = -\frac{\partial^2 U}{\partial {x_n}^2} t_m^w w_n$, $c = -\frac{\partial^2 U}{\partial {x_n}^2} t_m^w w_n$, $d = \frac{\partial^2 U}{\partial {x_n}^2} t_m^w w_n$

B. Cooperative Case – Non-Unitary Household

First Order Conditions

$$\lambda \frac{\partial U_m}{\partial t_m} + (1-\lambda)\frac{\partial U_n}{\partial x_n} w_m = 0$$

$$\frac{\partial U_n}{\partial t_n} + \frac{\partial U_n}{\partial x_n} w_n = 0$$

$$\lambda \frac{\partial U_m}{\partial x_m} - (1-\lambda)\frac{\partial U_n}{\partial x_n} = 0$$

$$\lambda\left(\frac{\partial U_m}{\partial t_m}\mu_m + \frac{\partial U_m}{\partial z}\frac{\partial z}{\partial t_m^h}\right) + (1-\lambda)\left(\frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_m^h}\right) = 0$$

$$\lambda\left(\frac{\partial U_m}{\partial z}\frac{\partial z}{\partial t_n^h}\right) + (1-\lambda)\left(\frac{\partial U_n}{\partial t_n}\mu_n + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right) = 0$$

$$\lambda\left(\frac{\partial U_m}{\partial t_k} + \frac{\partial U_m}{\partial z}\frac{\partial z}{\partial t_k}\right) + (1-\lambda)\left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right) = 0$$

Assumptions: (1) All goods separable in utility.
(2) No complementarities in household production.

Let *D* denote the determinant of the Hessian, and define its elements as

$$s_{11} = \lambda \frac{\partial^2 U_m}{\partial t_m{}^2} + (1-\lambda)\frac{\partial^2 U_n}{\partial x_n{}^2} w_m{}^2 < 0$$

$$s_{12} = (1-\lambda)\frac{\partial^2 U_n}{\partial x_n{}^2} w_m w_n < 0$$

$$s_{13} = -(1-\lambda)\frac{\partial^2 U_n}{\partial x_n{}^2} w_m > 0$$

$$s_{14} = \frac{\partial^2 U_m}{\partial t_m{}^2}\mu_m < 0$$

$$s_{22} = \frac{\partial^2 U_n}{\partial t_n{}^2} + \frac{\partial^2 U_n}{\partial x_n{}^2} w_n{}^2 < 0$$

$$s_{23} = -\frac{\partial^2 U_n}{\partial x_n{}^2} w_n > 0$$

$$s_{25} = \frac{\partial^2 U_n}{\partial t_n{}^2}\mu_n < 0$$

$$s_{33} = \lambda \frac{\partial^2 U_m}{\partial x_m^{\ 2}} + (1-\lambda)\frac{\partial^2 U_n}{\partial x_n^{\ 2}} < 0$$

$$s_{44} = \lambda \left[\frac{\partial^2 U_m}{\partial t_m^{\ 2}} \mu_m^{\ 2} + \frac{\partial^2 U_m}{\partial z^2}\left(\frac{\partial z}{\partial t_m^h}\right)^2 + \frac{\partial U_m}{\partial z}\frac{\partial^2 z}{\partial t_m^{h\,2}} \right] + (1-\lambda)\left[\frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_m^h}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_m^{h\,2}} \right] < 0$$

$$s_{45} = \lambda \left(\frac{\partial^2 U_m}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_n^h} \right) + (1-\lambda)\left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_n^h} \right) < 0$$

$$s_{46} = \lambda \left(\frac{\partial^2 U_m}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_k} \right) + (1-\lambda)\left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_k} \right) < 0$$

$$s_{55} = \lambda \left[\frac{\partial^2 U_m}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial U_m}{\partial z}\frac{\partial^2 z}{\partial t_n^{h\,2}} \right] + (1-\lambda)\left[\frac{\partial^2 U_n}{\partial t_n^{\ 2}} \mu_n^{\ 2} + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^{h\,2}} \right] < 0$$

$$s_{56} = \lambda \left(\frac{\partial^2 U_m}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k} \right) + (1-\lambda)\left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k} \right) < 0$$

$$s_{66} = \lambda \left[\frac{\partial^2 U_m}{\partial t_k^{\ 2}} + \frac{\partial^2 U_m}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial U_m}{\partial z}\frac{\partial^2 z}{\partial t_k^{\ 2}} \right] + (1-\lambda)\left[\frac{\partial^2 U_n}{\partial t_k^{\ 2}} + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k^{\ 2}} \right] < 0$$

$$s_{15} = s_{16} = s_{24} = s_{26} = s_{34} = s_{35} = s_{36} = 0$$

$$\left.\frac{dt_m^h}{dw_m}\right|^{compensated} = \frac{1}{D}\ [((s25s33s56s46 - s25s33s66s45)s12 + (-s25s56s23s46 + s25s66s23s45)s13$$
$$+ (s66s33s25^2 + s33s56^2s22 - s33s66s55s22 - s56^2s23^2 + s66s23^2s55)s14)e]$$

$$\left.\frac{dt_n^h}{dw_m}\right|^{compensated} = \frac{1}{D}\ [((-s25s33s46^2 + s25s33s66s44)s12 + (s25s23s46^2 - s25s23s66s44)s13$$
$$+ (-s23^2s66s45 + s33s66s45s22 + s46s23^2s56 - s46s33s56s22)s14)e]$$

$$\left.\frac{dt_k}{dw_m}\right|^{compensated} = \frac{1}{D}\ [((s25s33s45s46 - s25s33s56s44)s12 + (-s25s23s45s46 + s25s23s56s44)s13 +$$
$$(-s46s23^2s55 + s23^2s56s45 - s33s25^2s46 + s46s33s55s22 - s33s56s45s22)s14)e]$$

$$\frac{dt_n^h}{dt_m^h} < 0\,,\ \frac{dt_k}{dt_m^h}\ ambiguous$$

where $e = -(1-\lambda)\dfrac{\partial U_n}{\partial x_n}$

$$\left.\frac{dt_n^h}{dw_m}\right|^{income}_{t_m^h} = \frac{1}{D}\ [((s12s25s33 - s13s25s23)s66f + (-s11s25s33 + s13^2s25)s66g + (s11s25s23 -$$
$$s12s25s13)s66i]$$

$$\left.\frac{dt_k}{dw_m}\right|^{income}_{t_m^h} = \frac{1}{D}\ [((-s12s25s33 + s13s25s23)s56f + (s11s25s33 - s13^2s25)s56g + (-s11s25s23 +$$
$$s12s25s13)s56i]$$

$$\left.\frac{dt_n^h}{dw_m}\right|^{income}_{t_m^h} > 0\,,\ \left.\frac{dt_k}{dw_m}\right|^{income}_{t_m^h} < 0$$

where $f = -(1-\lambda)\dfrac{\partial^2 U_n}{\partial {x_n}^2}t_m^w w_n$, $g = -\dfrac{\partial^2 U_n}{\partial {x_n}^2}t_m^w w_n$, $i = (1-\lambda)\dfrac{\partial^2 U_n}{\partial {x_n}^2}t_m^w w_n$

$$\left.\frac{dt_n^h}{d\lambda}\right|_{t_m^h} = \frac{1}{D}\ [(s12s25s33 - s13s25s23)s66j + (-s12s25s13 + s11s25s23)s66k + (s11s22s33 - s11s23^2 - s22s13^2 - s12^2s33 + 2s12s23s13)s66m + (-2s12s23s13 - s11s22s33 + s12^2s33 + s11s23^2 + s22s13^2)s56n]$$

$$\left.\frac{dt_k}{d\lambda}\right|_{t_m^h} = \frac{1}{D}\ [(-s12s33 + s13s23)s25s56j + (s12s25s13 - s11s25s23)s56k + (-s11s22s33 + s11s23^2 + s22s13^2 + s12^2s33 - 2s12s23s13)s56m + ((s11s22s33 - s11s23^2 + 2s12s23s13 - s12^2s33 - s13^2s22)s55 + (-s11s33 + s13^2)s25^2)n]$$

$$\left.\frac{dx_n}{d\lambda}\right|_{t_m^h} = \frac{1}{D}\ [((s12s23s55 - s13s22s55 + s13s25^2)s66 + (-s12s23 + s13s22)s56^2)j + ((s11s22s55 - s12^2s55 - s11s25^2)s66 + (-s11s22 + s12^2)s56^2)k + (s11s25s23 - s12s25s13)s66m + (s12s25s13 - s11s25s23)s56n]$$

$$\left.\frac{dt_n^h}{d\lambda}\right|_{t_m^h} > 0,\ \left.\frac{dt_k}{d\lambda}\right|_{t_m^h} < 0,\ \left.\frac{dx_n}{d\lambda}\right|_{t_m^h} > 0 \ \text{ if } \left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right) \geq 0$$

where $j = \frac{1}{\lambda^2}\frac{\partial U_n}{\partial x_n} w_m, k = -\frac{1}{\lambda^2}\frac{\partial U_n}{\partial x_n},\ m = -\frac{1}{\lambda^2}\left(\frac{\partial U_n}{\partial t_n}\mu_n + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right)$ and

$n = -\frac{1}{\lambda^2}\left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right)$

C. Non-Cooperative Case

First Order Conditions

$$\left(\frac{\partial U_n}{\partial t_n}+\frac{\partial U_n}{\partial x_n}w_n\right)-\delta q\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)-\frac{\partial q}{\partial x_n}w_n\delta(U_n-U_n^{'})=0,$$
$$\left(\frac{\partial U_n}{\partial t_n}\mu_n+\frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right)-\left(\frac{\partial q}{\partial t_n^h}+\frac{\partial q}{\partial z}\frac{\partial z}{\partial t_n^h}\right)\delta(U_n-U_n^{'})=0 \text{ and}$$
$$\left(\frac{\partial U_n}{\partial t_k}+\frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right)-\left(\frac{\partial q}{\partial t_k}+\frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)\delta(U_n-U_n^{'})=0$$

Assumptions: (1) All goods separable in utility.
(2) No complementarities in household production.
(3) No cross-good effects in $q(\cdot)$.
(4) $\delta q\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)+\frac{\partial q}{\partial x_n}w_n\delta(U_n-U_n^{'})>0$
(5) $\left(\frac{\partial q}{\partial t_k}+\frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)>0$
(6) $\sigma_{23}<0$

Let Δ denote the determinant of the Hessian, and define its elements as

$$\sigma_{11}=\frac{\partial^2 U_n}{\partial t_n^{\,2}}+\frac{\partial^2 U_n}{\partial x_n^{\,2}}w_n^{\,2}-\delta q\left(\frac{\partial^2 U_n}{\partial x_n^{\,2}}w_n^{\,2}-\frac{\partial^2 U_n^{'}}{\partial x_n^{\,2}}w_n^{\,2}\right)-\frac{\partial^2 q}{\partial x_n^{\,2}}w_n^2\delta(U_n-U_n^{'})-2\frac{\partial q}{\partial x_n}w_n\delta\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)<0$$
$$\sigma_{12}=\frac{\partial^2 U_n}{\partial t_n^{\,2}}\mu-\left(\frac{\partial q}{\partial t_n^h}+\frac{\partial q}{\partial z}\frac{\partial z}{\partial t_n^h}\right)\delta\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)<0$$

$$\sigma_{13} = -\left(\frac{\partial q}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)\delta\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) > 0$$

$$\sigma_{22} = \left(\frac{\partial^2 U_n}{\partial t_n^{\ 2}}\mu_n^{\ 2} + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^{h2}}\right) - \left[\frac{\partial^2 q}{\partial t_n^{h2}} + \frac{\partial^2 q}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial q}{\partial z}\left(\frac{\partial^2 z}{\partial t_n^{h2}}\right)\right]\delta(U_n - U_n^{'}) < 0$$

$$\sigma_{23} = \left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k}\right) - \left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k}\right)\delta(U_n - U_n^{'}) < 0$$

$$\sigma_{33} = \left[\frac{\partial^2 U_n}{\partial t_c^{\ 2}} + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k^{\ 2}}\right] - \left[\frac{\partial^2 q}{\partial t_k^{\ 2}} + \frac{\partial^2 q}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k^{\ 2}}\right]\delta(U_n - U_n^{'}) < 0$$

$$\frac{dt_n^w}{ds^c} = \frac{1}{\Delta}\left\{\left[-\frac{\partial^2 U_n}{\partial x_n^{\ 2}} w_n(1-\delta q) + \frac{\partial q}{\partial x_n} w_n \delta\frac{\partial U_n}{\partial x_n}\right](\sigma_{22}\sigma_{33} - \sigma_{23}^{\ 2}) - \left(\frac{\partial q}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_m^h}\right)\delta\frac{\partial U_n}{\partial x_n}(\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23})\right.$$
$$\left. + \left(\frac{\partial q}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)\delta\frac{\partial U_n}{\partial x_n}(\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})\right\} < 0$$

$$\frac{dt_n^h}{ds^c} = \frac{1}{\Delta}\left\{\left[\frac{\partial^2 U_n}{\partial x_n^{\ 2}} w_n(1-\delta q) - \frac{\partial q}{\partial x_n} w_n \delta\frac{\partial U_n}{\partial x_n}\right](\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) + \left(\frac{\partial q}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_m^h}\right)\delta\frac{\partial U_n}{\partial x_n}(\sigma_{11}\sigma_{33} - \sigma_{13}^{\ 2})\right.$$
$$\left. - \left(\frac{\partial q}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)(\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right\} > 0$$

$$\frac{dt_k}{ds^c} = \frac{1}{\Delta}\left\{\left[-\frac{\partial^2 U_n}{\partial x_n^{\ 2}} w_n(1-\delta q) + \frac{\partial q}{\partial x_n} w_n \delta\frac{\partial U_n}{\partial x_n}\right](\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}) - \left(\frac{\partial q}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_m^h}\right)\delta\frac{\partial U_n}{\partial x_n}(\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right.$$
$$\left. + \left(\frac{\partial q}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)(\sigma_{11}\sigma_{22} - \sigma_{12}^{\ 2})\right\} < 0$$

$$\frac{dt_n^w}{dt_n^{h^c}} = \frac{1}{\Delta}\left\{\delta \frac{\partial q}{\partial t_n^{h^c}}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right)(\sigma_{22}\sigma_{33} - \sigma_{23}{}^2) - \frac{\partial^2 q}{\partial t_n^h \partial t_n^{h^c}} \delta(U_n - U_n^{'})(\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23})\right\}$$

$$\frac{dt_n^h}{dt_n^{h^c}} = \frac{1}{\Delta}\left\{\delta \frac{\partial q}{\partial t_n^{h^c}}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right)[-(\sigma_{12}\sigma_{33} - \sigma_{23}\sigma_{13})] + \frac{\partial^2 q}{\partial t_n^h \partial t_n^{h^c}} \delta(U_n - U_n^{'})(\sigma_{11}\sigma_{33} - \sigma_{13}{}^2)\right\}$$

$$\frac{dt_k}{dt_n^{h^c}} = \frac{1}{\Delta}\left\{\delta \frac{\partial q}{\partial t_n^{h^c}}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right)(\sigma_{12}\sigma_{23} - \sigma_{22}\sigma_{13}) - \frac{\partial^2 q}{\partial t_n^h \partial t_n^{h^c}} \delta(U_n - U_n^{'})(\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right\}$$

$$\frac{dt_n^w}{dt_k{}^c} = \frac{1}{\Delta}\left\{\delta \frac{\partial q}{\partial t_k{}^c}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right)(\sigma_{22}\sigma_{33} - \sigma_{23}{}^2) + \frac{\partial^2 q}{\partial t_k \partial t_k{}^c} \delta(U_n - U_n^{'})(\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})\right\}$$

$$\frac{dt_n^h}{dt_k{}^c} = \frac{1}{\Delta}\left\{\delta \frac{\partial q}{\partial t_k{}^c}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right)[-(\sigma_{12}\sigma_{33} - \sigma_{23}\sigma_{13})] - \frac{\partial^2 q}{\partial t_k \partial t_k{}^c} \delta(U_n - U_n^{'})(\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right\}$$

$$\frac{dt_k}{dt_k{}^c} = \frac{1}{\Delta}\left\{\delta \frac{\partial q}{\partial t_k{}^c}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right)(\sigma_{12}\sigma_{23} - \sigma_{22}\sigma_{13}) + \frac{\partial^2 q}{\partial t_k \partial t_k{}^c} \delta(U_n - U_n^{'})(\sigma_{11}\sigma_{22} - \sigma_{12}{}^2)\right\}$$

$$\frac{dt_n^w}{d\omega_q} = \frac{1}{\Delta}\left\{\left[\delta\frac{\partial q}{\partial\omega_q}\left(\frac{\partial U_n}{\partial x_n}w_n - \frac{\partial U_n^{'}}{\partial x_n}w_n\right) + \frac{\partial^2 q}{\partial x_n\partial\omega_q}w_n\delta(U_n - U_n^{'})\right](\sigma_{22}\sigma_{33} - \sigma_{23}{}^2) - \left[\left(\frac{\partial^2 q}{\partial t_n^h\partial\omega_q} + \frac{\partial^2 q}{\partial z\partial\omega_q}\frac{\partial z}{\partial t_m^h}\right)\delta(U_n - U_n^{'})\right](\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23})\right.$$

$$\left. + \left[\left(\frac{\partial^2 q}{\partial t_k\partial\omega_q} + \frac{\partial^2 q}{\partial z\partial\omega_q}\frac{\partial z}{\partial t_k}\right)\delta(U_n - U_n^{'})\right](\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})\right\} < 0$$

$$\frac{dt_n^h}{d\omega_q} = \frac{1}{\Delta}\left\{-\left[\delta\frac{\partial q}{\partial\omega_q}\left(\frac{\partial U_n}{\partial x_n}w_n - \frac{\partial U_n^{'}}{\partial x_n}w_n\right) + \frac{\partial^2 q}{\partial x_n\partial\omega_q}w_n\delta(U_n - U_n^{'})\right](\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) + \left[\left(\frac{\partial^2 q}{\partial t_n^h\partial\omega_q} + \frac{\partial^2 q}{\partial z\partial\omega_q}\frac{\partial z}{\partial t_n^h}\right)\delta(U_n - U_n^{'})\right](\sigma_{11}\sigma_{33} - \sigma_{13}{}^2)\right.$$

$$\left. - \left[\left(\frac{\partial^2 q}{\partial t_k\partial\omega_q} + \frac{\partial^2 q}{\partial z\partial\omega_q}\frac{\partial z}{\partial t_k}\right)\delta(U_n - U_n^{'})\right](\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right\} > 0$$

$$\frac{dt_k}{d\omega_q} = \frac{1}{\Delta}\left\{\left[\delta\frac{\partial q}{\partial\omega_q}\left(\frac{\partial U_n}{\partial x_n}w_n - \frac{\partial U_n^{'}}{\partial x_n}w_n\right) + \frac{\partial^2 q}{\partial x_n\partial\omega_q}w_n\delta(U_n - U_n^{'})\right](\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}) - \left[\left(\frac{\partial^2 q}{\partial t_n^h\partial\omega_q} + \frac{\partial^2 q}{\partial z\partial\omega_q}\frac{\partial z}{\partial t_n^h}\right)\delta(U_n - U_n^{'})\right](\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right.$$

$$\left. + \left[\left(\frac{\partial^2 q}{\partial t_k\partial\omega_q} + \frac{\partial^2 q}{\partial z\partial\omega_q}\frac{\partial z}{\partial t_k}\right)\delta(U_n - U_n^{'})\right](\sigma_{11}\sigma_{22} - \sigma_{12}{}^2)\right\} < 0$$

$$\frac{dt_n^w}{d\omega_x} = \frac{1}{\Delta}\left\{\delta\frac{\partial q}{\partial\omega_x}\left(\frac{\partial U_n}{\partial x_n}w_n - \frac{\partial U_n^{'}}{\partial x_n}w_n\right) + \frac{\partial^2 q}{\partial x_n\partial\omega_x}w_n\delta(U_n - U_n^{'})\right\}(\sigma_{22}\sigma_{33} - \sigma_{23}{}^2) < 0$$

$$\frac{dt_n^h}{d\omega_x} = \frac{1}{\Delta}\left\{\delta\frac{\partial q}{\partial\omega_n}\left(\frac{\partial U_n}{\partial x_n}w_n - \frac{\partial U_n^{'}}{\partial x_n}w_n\right) + \frac{\partial^2 q}{\partial x_n\partial\omega_x}w_n\delta(U_n - U_n^{'})\right\}[-(\sigma_{12}\sigma_{33} - \sigma_{23}\sigma_{13})] > 0$$

$$\frac{dt_k}{d\omega_x} = \frac{1}{\Delta}\left\{\delta\frac{\partial q}{\partial\omega_x}\left(\frac{\partial U_n}{\partial x_n}w_n - \frac{\partial U_n^{'}}{\partial x_n}w_n\right) + \frac{\partial^2 q}{\partial x_n\partial\omega_x}w_n\delta(U_n - U_n^{'})\right\}(\sigma_{12}\sigma_{23} - \sigma_{22}\sigma_{13}) < 0$$

$$\frac{dt_n^w}{d\omega_z}=\frac{1}{\Delta}\left\{\delta\frac{\partial q}{\partial\omega_z}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{22}\sigma_{33}-\sigma_{23}{}^2)-\frac{\partial^2 q}{\partial z\partial\omega_z}\delta(U_n-U_n^{'})\left[\frac{\partial z}{\partial t_n^h}(\sigma_{12}\sigma_{33}-\sigma_{13}\sigma_{23})-\frac{\partial z}{\partial t_k}(\sigma_{12}\sigma_{23}-\sigma_{13}\sigma_{22})\right]\right\}\text{ amb.}$$

$$\frac{dt_n^h}{d\omega_z}=\frac{1}{\Delta}\left\{\delta\frac{\partial q}{\partial\omega_z}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)[-(\sigma_{12}\sigma_{33}-\sigma_{23}\sigma_{13})]+\frac{\partial^2 q}{\partial z\partial\omega_z}\delta(U_n-U_n^{'})\left[\frac{\partial z}{\partial t_n^h}(\sigma_{11}\sigma_{33}-\sigma_{13}{}^2)-\frac{\partial z}{\partial t_k}(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})\right]\right\}\text{ amb.}$$

$$\frac{dt_k}{d\omega_z}=\frac{1}{\Delta}\left\{\delta\frac{\partial q}{\partial\omega_z}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{12}\sigma_{23}-\sigma_{22}\sigma_{13})-\frac{\partial^2 q}{\partial z\partial\omega_z}\delta(U_n-U_n^{'})\left[\frac{\partial z}{\partial t_n^h}(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})-\frac{\partial z}{\partial t_k}(\sigma_{11}\sigma_{22}-\sigma_{12}{}^2)\right]\right\}\text{ amb.}$$

$$\frac{dt_n^w}{d\mu}=\frac{1}{\Delta}\left(-\frac{\partial U_n}{\partial t_n}-\frac{\partial^2 U_n}{\partial t_n{}^2}t_n^h\mu\right)[-(\sigma_{12}\sigma_{33}-\sigma_{13}\sigma_{23})]>0$$

$$\frac{dt_m^h}{d\mu}=\frac{1}{\Delta}\left(-\frac{\partial U_n}{\partial t_n}-\frac{\partial^2 U_n}{\partial t_n{}^2}t_n^h\mu\right)(\sigma_{11}\sigma_{33}-\sigma_{13}{}^2)<0$$

$$\frac{dt_k}{d\mu}=\frac{1}{\Delta}\left(-\frac{\partial U_n}{\partial t_n}-\frac{\partial^2 U_n}{\partial t_n{}^2}t_n^h\mu\right)[-(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})]$$

$$\begin{aligned}\frac{dt_n^w}{d\tau_n}=\frac{1}{\Delta}\Bigg\{&\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_n^h}+\frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_n^h\partial\tau_n}\right)\delta(U_n-U_n^{'})-\frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^h\partial\tau_n}-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_n^h}\right][-(\sigma_{12}\sigma_{33}-\sigma_{13}\sigma_{23})]\\&+\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_k}\right)\delta(U_n-U_n^{'})-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_k}\right](\sigma_{12}\sigma_{23}-\sigma_{13}\sigma_{22})\Bigg\}<0\end{aligned}$$

$$\begin{aligned}\frac{dt_n^h}{d\tau_n}=\frac{1}{\Delta}\Bigg\{&\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_n^h}+\frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_n^h\partial\tau_n}\right)\delta(U_n-U_n^{'})-\frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^h\partial\tau_n}-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_n^h}\right](\sigma_{11}\sigma_{33}-\sigma_{13}{}^2)\\&-\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_k}\right)\delta(U_n-U_n^{'})-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial\tau_n}\frac{\partial z}{\partial t_k}\right](\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})\Bigg\}>0\end{aligned}$$

$$\frac{dt_k}{d\tau_n}=\frac{1}{\Delta}\left\{\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h}+\frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_n^h\partial \tau_n}\right)\delta(U_n-U_n^{'})-\frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^h\partial \tau_n}-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h}\right][-(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})]\right.$$

$$\left.+\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right)\delta(U_n-U_n^{'})-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right](\sigma_{11}\sigma_{22}-\sigma_{12}{}^2)\right\}<0$$

$$\frac{dt_n^w}{d\tau_k}=\frac{1}{\Delta}\left\{\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}+\frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k\partial \tau_k}\right)\delta(U_n-U_n^{'})-\frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k\partial \tau_k}-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}\right](\sigma_{12}\sigma_{23}-\sigma_{13}\sigma_{22})\right.$$

$$\left.-\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right)\delta(U_n-U_n^{'})-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right](\sigma_{12}\sigma_{33}-\sigma_{13}\sigma_{23})\right\}>0$$

$$\frac{dt_n^h}{d\tau_k}=\frac{1}{\Delta}\left\{\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}+\frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k\partial \tau_k}\right)\delta(U_n-U_n^{'})-\frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k\partial \tau_k}-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}\right][-(\sigma_{11}\sigma_{23}-\sigma_{13}\sigma_{12})]<0\right.$$

$$\left.+\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right)\delta(U_n-U_n^{'})-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right](\sigma_{11}\sigma_{33}-\sigma_{13}{}^2)\right\}$$

$$\frac{dt_k}{d\tau_k}=\frac{1}{\Delta}\left\{\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}+\frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k\partial \tau_k}\right)\delta(U_n-U_n^{'})-\frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k\partial \tau_k}+\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}\right](\sigma_{11}\sigma_{22}-\sigma_{12}{}^2)\right.$$

$$\left.-\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right)\delta(U_n-U_n^{'})-\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right](\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})\right\}>0$$