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Designing Countercyclical and Risk Based Aggregate Deposit Insurance Premia

Abstract

This paper proposes an aggregate deposit insurance premium design that is risk-based in the sense that the premium structure ensures the deposit insurance system has a target of survival over the longer term. Such a premium system naturally exceeds the actuarily fair value and leads to a growth in the insurance fund over time. The proposed system builds in a swap in premia that reduces premia when fund size exceeds a threshold. In addition, we build in a swap contract that trades premia in good times for relief in bad times.

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1 Introduction

The Federal Deposit Insurance Corporation Act of 1991 mandates the adoption of a risk based deposit insurance premium structure for US Banks. Basel II proposal moved towards requiring bank capital levels to be risk based. Recently, the Federal Deposit Insurance Reform Act of 2005 permits the FDIC to charge every bank a premium based on risk, provide initial assessment credits to banks that helped to build up the insurance funds, and require the FDIC to pay rebates if the ratio of insurance fund size to insured deposits (reserve ratio) exceeds certain thresholds. While theoretically very appealing, concerns have been raised about the procyclical adverse impact of risk based premia and capital requirements (Blinder and Wescott, 2001; Allen and Saunders, 2004). Banking organizations are required to pay higher premia and hold greater capital in economic downturns, thereby aggravating the effects of economic recessions.

This paper proposes an aggregate premium policy that is countercyclical by design and is nonetheless founded on risk based principles. The basic idea is to build into the premium system a swap contract that trades premia in good times for relief in bad times. In addition, the proposed design does not allow the deposit insurance fund size to grow excessively in a prolonged economic boom. This feature is engineered by also incorporating a premium reduction swap when the fund size exceeds a target level. The design therefore builds into the aggregate premium policy an elasticity for both downturns and fund growth, with premiums taking a percentage reduction in response to the depth of the downturn and the excess fund size.

The target fund size, aggregate premium level and rebate structures are all risk based by ensuring that the deposit insurance system has a high probability of survival over the longer term. We first establish a benchmark case where there is no premium rebates. In such a system we determine the target fund size and aggregate premium level that ensures a long term fund survival. We then evaluate the effects of altering rebates and premium levels and fund-size targets in the neighborhood of the benchmark level on the probability of long term fund survival. We report the trade-offs that are implicit between these policies when we enforce neutrality of the long term survival probability.

We determine the long term survival probability by simulating the operation of the system through time. For the determination of these basic trade-offs we consider a time homogeneous formulation of the risks involved and we essentially perform all calculations in present value terms by considering a zero interest rate economy. The risks facing the system are the time series of losses on which the system has to payout and we build for this purpose a model for the aggregate loss distribution. The model entails three uncertain components. They are the number of loss events (bank failures), the asset size of the failing banks, and the loss rate given default.

The model employs the unconditional distributions of these events. We model the number of loss events by a Poisson process with a constant arrival rate. For the distribution of asset sizes, we analyze the data on bank asset sizes for US banks over the years 2000-2003. We confirm that the Frechet distribution provides a statistically good fit for the data. With respect to loss rates, we analyze the data on loss rates experienced by the Federal Deposit Insurance Corporation over the period 1984-2002 on 1508 bank failures. We show that the Weibull model provides a statistically good fit to describe the historical loss experience.

We build these loss components into a simulation of the effects of alternative premium policies and evaluate the probability of long term fund survival resulting from the adoption of these risk based and countercyclical premium policies. The final result demonstrates the trade-offs between the various policy dimensions for a given target long term survival probability. The specific policy dimensions are the downturn rebate, the rebate for excessive growth of the fund, and the level of aggregate premiums and target fund size.

The outline of the paper is as follows. Section 2 presents the design of the counteryclical and risk based premium system. In Section 3 we present the results on modeling the data on asset sizes and loss rates. In Section 4 we describe the simulation of the long term survival probability and establish the benchmarks for fund size and aggregate premiums. Section 5 constructs the trade-off table. Section 6 concludes.

2 The Aggregate Premium System

The aggregate premium system we consider has three risk based components. They are a flat rate, rebate enhancements based on the fund size, and the level of aggregate losses due to bank failures. The premium is envisaged as charged at the end of the year, when the level of losses is known. The risk considerations in determining premium policy focuses on the financial health of the insurance fund. The objective is to enforce a premium structure that is not excessive but yet ensures the viability of the insurance system. Specifically, in this study, we target a 95% probability of surviving 10 years. Hence, we seek to minimize premiums that are countercyclical and yet meet such a target. Such probability target allows taxpayers to bear some of the default risk of the insurance fund. As indicated by Blinder and Wescott (2001), "reducing the taxpayers' potential exposure all the way to zero is not the appropriate goal of policy."

For the analysis of the risks involved we adopt a discrete time model with annual periods denoted by $n = 0, \cdots, N$. We also work in present value terms or equivalently consider a zero interest rate economy. The flat aggregate premium is κ in billions of dollars per year. Denote by C_n the size of the fund, in billions of dollars, at the start of period n, and let L_n be the level of losses, in billions of dollars, paid out by the Insurance fund in period n. We introduce a premium rebate for fund sizes above a benchmark level C. The elasticity of premium rebate with respect to C_n exceeding C is β. We also introduce a rebate with elasticity γ with respect to the aggregate loss level L_n. The annual premium assessed at the end of the year n, P_n, is then

$$P_n = \kappa \left(\max \left(\frac{C_n}{C}, 1 \right) \right)^{-\beta} (1 + L_n)^{-\gamma} \tag{1}$$

For $C_n < C$ and $L_n = 0$ the premium set by equation (1) is the flat rate of κ billion dollars. In other words, κ is the zero-rebate premium. In states $C_n = C$ and $L_n = 0$, κ must be high enough to allow for rebates in bad states of the world. For a fund size that exceeds the benchmark level of C by 10% the premium that is rebated is $0.1\beta\kappa$ billion dollars. Furthermore, for a 10% increase in the level of losses the premiums rebated are $0.1\gamma(1 + L_n)$ billions of dollars.

We suppose the system starts out with the benchmark level for the fund size, of C, and we present the trade-offs in the flat rate κ, the rebate elasticities β, γ that are consistent with long term fund survival measured by a 95% target probability of surviving 10 years.

The only inflows into the fund are the premiums and the only outflows are losses. The fund size at the start of the next period is then given by

$$C_{n+1} = C_n + P_n - L_n \tag{2}$$

and the insurance fund is declared bankrupt when the fund size C_n reaches a minimal reserve level.

The generation of annual aggregate losses L_n and the operation of the system over time are as follows. We analyze the long term survival probabilities by simulating the annual losses for ten years. A random number M_n of failures each year generate the aggregate loss amount. Each of these failures has an associated asset size, A_k for failure by bank k, and loss rate l_k with the k^{th} loss amount being $A_k l_k$ and $L_n = \sum_{k=1}^{M_n} A_k l_k$. The asset sizes are drawn from a stable aggregate distribution of asset sizes, and likewise loss rates are drawn from a stable and independent distribution of loss rates. The loss arrivals are generated by a Poisson process with a constant arrival rate. The asset size and loss rate distributions employed in the study are developed in the next section.

3 The Risk Neutralization of Premia

One approach to set annual premiums is at their actuarially fair level or equal to the statistically expected loss level. The statistical expectation is a sound pricing principle provided one has a large number of independent contracts and the expectation is the realized cash flow period by period with high probability. For example with fire insurance one may sign up a large number of independent contracts and each year we collect in premiums the expected payout while the actual payout is a random outcome that by the law of large numbers is essentially equal to the total annual premium. Note here that we have a large number of independent contracts and we have collected all the annual premiums each year. As a result we have an essentially zero cash flow every year.

One may argue that such an actuarially fair approach is appropriate for deposit insurance premiums as we do have a large number of banks in the system. However, within any given year there may be a lot of correlation across bank failures and independence is accessed at best only over time. Suppose we have enough independence over ten years. In this case we must charge every bank the actuarially fair premium for this period today, with no premiums in the next ten years, and when we payout all the losses for ten year period, we will have an essentially zero cash flow to the system at the end of ten years. The procedure can be repeated every ten years. This is not an operational system as future premiums cannot be collected up front and the fire insurance analogy breaks down.

An alternative is to have a capital reserve that pays out losses in the possibly early bad years with a view to replenishing the capital from the collection of future premiums. The system then requires an initial capital level to ensure that the insurance fund is capable of functioning independently over time. If the contributors to the capital are the insured pool themselves then their premium must reflect a charge for contribution to capital over and above the expected loss that they are being covered for. Once we have a premium above expected value for whatever reason, we are involved in some form of risk neutralization. The contribution to capital is a cost of developing an effective and operable insurance system and is in this sense a part of the price of insurance. The charge for capital contribution plus the actuarial loss level is equal to an expected loss calculation under a specific change of probability and is therefore equivalent to a risk neutralization.

The extent of the capital contribution depends on how one defines a functional insurance system. For example one could target a 99% probability of surviving 30 years or just a 95% probability of surviving 10 years. Clearly the capital contribution in the former case is larger than the latter. In the current context we may conceptualize the situation as follows. Let C_{Ns} be the level of the fund at some distant time N on the scenario path s. Bear in mind that we allow C_{Ns} to be negative if losses dominate the fund size coupled with premium collections in the interim. Any system that we put in place will lead us to a random variable C_N with the outcomes $(C_{Ns}, s \in S)$, where S is the set of all scenarios. A functional insurance system must define what risk levels seen as exposure to negative fund sizes at time N are tolerable. Recent risk theory, has axiomatically characterized the definition of acceptable risk levels.

Artzner, Delbaen, Eber and Heath (1999) define acceptable cash flows as including all positive random variables, but permitting a limited exposure to some negative outcomes. The collection of all acceptable cash flows is modeled as cash flows that have a positive expected outcome under a sufficient number of simulated scenario tests. Each simulated scenario expectation computation is the calculation of $E^Q[C_N]$ for some test measure Q. Formally, cash flows C_N are acceptable just if

$$E^{Q_i}[C_N] \geq 0$$

for a collection of M test measures Q_i, $i = 1, \cdots, M$.

Clearly we wish to minimize the premium charge subject to the constraint of having an acceptable result. We may thus write the problem of determining the annual premium κ for N years as

$$
\begin{aligned}
&Min\ N\kappa \\
&S.T.\ E^{Q_i}[L] - N\kappa \leq 0,\ \ i = 1, \cdots, M
\end{aligned}
$$

The dual to this simple linear programming problem yields

$$
N\kappa = Max_{\lambda \geq 0} \sum_i \lambda_i E^{Q_i}[L]
$$

Hence, we see the aggregate premium as an expected loss computation using a measure in the convex hull of the measures defining acceptability. This is the form of risk neutralization induced by the objective of constructing functioning insurance systems.

In general it is quite difficult to envisage what the reasonable candidate test measures Q_i should be. Recent progress in the theory of acceptability has focused on law invariant measures of risk and acceptability, that is measures that depend solely on the probability distribution of the aggregate cash flow outcome. These law invariant risk measures have been characterized by Kusuoka (2001) as a form of weighted expected shortfall taken at all possible levels of VAR.

For the moment we ignore the expected shortfall and target a VAR like measure by ensuring that the probability of the fund getting below a threshold over a specified number of years is sufficiently low, or equivalently the probability of staying above the threshold is sufficiently high. Specifically we target a 95% probability of staying above a threshold for 10 years.

We now recognize that organizing premia to meet long term survival targets will force a premium level above the actuarially fair level and as a result the fund size will grow over time. It is then imperative to build into the system a capital rebate feature that reduces premia when the fund size crosses a threshold. We build into the system investigated such automatic premium rebate mechanisms.

4 Asset Size and Loss Rate Distributions

The asset sizes that are relevant are those of all potential banks that are subject to failure in any given year. The loss rates are those that have been experienced in past bank failures. For the US banking system, the distribution of asset sizes has a characteristic property of a substantial number of banks that have assets that are an order of magnitude above the modal or most likely asset size. Such a fat tailed distribution can be well modeled by the parametric class of Frechet distributions. This distribution has two parameters, a scale parameter c_F and a shape parameter a_F with the cumulative distribution function $F(A; c_F, a_F)$ given by

$$F\left(A; c_F, a_F\right) = \exp\left(-\left(\frac{A}{c_F}\right)^{-a_F}\right).$$

The associated density is $f(A; c_F, a_F)$ and

$$f(A; c_F, a_F) = \exp\left(-\left(\frac{A}{c_F}\right)^{-a_F}\right) \frac{a_F c_F^{a_F}}{A^{1+a_F}} \tag{3}$$

and the tail of the distribution falls at rate $1 + a_F$ with the consequence that moments exist only for orders less than a_F. Hence the mean, μ_F, is finite for $a_F > 1$ and the variance, σ_F^2 is finite for $a_F > 2$ in which case

$$\begin{aligned}
\mu_F &= c_F \Gamma\left(1 - \frac{1}{a_F}\right) \\
\sigma_F^2 &= c_F^2 \left(\Gamma\left(1 - \frac{2}{a_F}\right) - \Gamma\left(1 - \frac{1}{a_F}\right)^2\right)
\end{aligned}$$

The Frechet distribution has a mode A_m below c_F at the point

$$A_m = c_F \left(1 + \frac{1}{a_F}\right)^{-\frac{1}{a_F}}$$

that reflects a positive most likely asset size and yet it has a long tail with substantial probability at large sizes as the density decays at a power law. In this regard it is particularly suited for describing the distribution of asset sizes in a population of many small banks with a few very large ones.

For the loss rate distribution we consider the parametric class of the Weibull distribution. Loss rates are essentially bounded variables for which all moments are finite. This is true for the Weibull family. This family also has two parameters, a scale parameter c_W,and a shape parameter a_W with the cumulative distribution function $G(L; c_W, a_W)$ given by

$$G(L; c_W, a_W) = 1 - \exp\left(-\left(\frac{L}{c_W}\right)^{a_W}\right).$$

The associated density is $g(L; c_W, a_W)$ and

$$g(L; c_W, a_W) = \exp\left(-\left(\frac{L}{c_W}\right)^{a_W}\right)\frac{a_w L^{a_W - 1}}{c_W^{a_W}} \tag{4}$$

The mean, μ_W and variance, σ_W^2 are given by

$$\begin{aligned}
\mu_W &= c_W \Gamma\left(1 + \frac{1}{a_W}\right) \\
\sigma_W^2 &= c_W^2\left(\Gamma\left(1 + \frac{2}{a_W}\right) - \Gamma\left(1 + \frac{1}{a_W}\right)^2\right)
\end{aligned}$$

For $a_W < 1$ this density has a mode at zero representing a most likely loss rate of zero. However, for the case $a_W > 1$ we have a modal loss level L_m below c_W of

$$L_m = c_W\left(1 - \frac{1}{a_W}\right)^{\frac{1}{a_W}}.$$

Furthermore, in this case the probability in the upper tail decreases at a rate that is faster than exponential which makes loss rates near unity relatively uncommon. The shape parameter of the

Weibull distribution, a_W parametrizes the behavior of the hazard rate for losses. The hazard rate is the relative probability of a large loss rate to an even larger loss rate. When hazard rates are increasing it gets more and more difficult to get to higher and higher loss rate levels. For the Weibull, with $a_W < 1$ we have decreasing hazard rates while for $a_W > 1$ we have increasing hazard rates.

For the distribution of loss rates we anticipate a positive mode, with an increasing hazard rate as all attempts are being made to limit losses. Hence the Weibull model with shape parameter above unity is appropriate. We also note that the Weibull model would be inappropriate for asset sizes as it has a fat tail only when its mode is zero while asset sizes have a fat tail with a positive mode. Similarly, the Frechet model is inappropriate for loss rates as it would generate a large number of loss rates above unity. Our empirical tests on the data confirm these conjectures.

Other distributional candidates may also be considered from the prior literature (Madan and Unal, 2004; Unal, Madan, and Guntay (2004), Kuritzkes, Schuermann, and Weiner, 2004). We also provide additional empirical tests of these alternatives with respect to our proposed choices and confirm the adequacy of the model we adopt.

4.1 Empirical Tests of the Distributional Models

For the distribution of asset sizes we use the asset sizes of 8694 banks in the US from the Call Report Data for the year 2000. The loss rate data come from the failed bank data base maintained at the FDIC for the period $1984-2000$. The number of failures were 1505 of which 32 had a zero loss rate. Our analysis uses the 1473 failed banks with positive loss rates. Table 1 summarizes the time series loss experience of the FDIC for the $1984-2000$ period.[1] It shows the yearly estimated losses as a percentage of the total assets of the failed banks together with the number of failures for six size categories. Three trends are observable. First, number of bank failures decline as bank asset size increases. For example, only eight banks over \$5 billion asset size failed during

[1] The data on breakdown of losses by asset size categories for 1984 are not available in publicly available FDIC documents,

the sample period. Second, as asset size increases loss rates decline. For example, while average loss rate for the smallest asset size group is about 25%, for the largest size group this average percentage declines to about 8%. Finally, after 1992, there is a notable decline in the number of bank failures.

The summary statistics on asset sizes and loss rates on failed banks are reported in Table 2. We observe that for the asset size the mean is substantially above the median and in fact also above the upper quartile, suggestive of a highly skewed and fat tailed distribution. This property is also reflected in the large standard deviation. Furthermore the last percentile relative is 51 times the upper quartile. Hence, the Frechet distribution appears to be an appropriate choice.

We observe that the average loss rate for the $1984-2000$ period is 21.1% of the assets. We realize that this rate and the distribution of the loss rates obtained from the $1984-2000$ period may not reflect the loss distribution faced by the FDIC in the next decade. One important consideration is the prompt corrective action (PCA) provision of the FDICIA, which requires regulatory intervention in advance of insolvency. Such mandate can substantially reduce expected costs (Blinder and Wescott, 2001). However, we use this period to allow for the possibility of adverse macro shocks experienced in the $1980s$.

Table 2 shows that, the mean and median loss rates are fairly close with the mean in the interquartile range. In addition, the last percentile is well below the unit loss rate and this observation suggests a substantially thinner upper tail. Thus, the Weibull model with a shape parameter above unity, appears a reasonable choice. The differences between the loss rate and asset size data sets are quite marked and provides the early indication that it is not likely that the two data sets come from the same distributional model.

Nevertheless, both the Frechet (equation 3) and Weibull (equation 4) models are estimated by maximum likelihood on the asset size data scaled to \$10 billions, for the 8649 banks in the year 2000. The parameter estimates for the Frechet are $a_F = 0.94002$, $c_F = 0.005154$ and the estimates for the Weibull are $a_W = 0.5426$, $c_W = 0.0204$. For graphical convenience Figure (1) plots the histogram of the binned data in steps of $10 million up to $500 million. This segment contains 90% of the data.

We observe that the Frechet model fits the data better. It picks up the mode and the long tail quite accurately. The Weibull on the other hand tries to get the long tail and as a consequence is forced to place the mode at zero. The quality of the improvement of the Frechet over the Weibull model is confirmed by Chi Square tests performed in the range of cells with more than 10 observations that go up to asset sizes of $700 million. The Frechet model could be improved upon in the smaller asset sizes below $250 million. For the range from $250 million to $700 million we have 43 degrees of freedom with the Frechet chi square statistic of 56.69 while the corresponding Weibull value is 416.67. The respective $p - values$ are .0787 for the Frechet and zero for the Weibull.

For the loss rate data in addition to the Frechet and Weibull models we employ three other distributions that have been used to describe loss distributions in the literature. These are the Gaussian with parameters μ_G, σ_G, the Beta distribution with two parameters α, β and the logit-normal with parameters μ_L, σ_L. The results are presented in Table 3.

Table 3 shows that the Beta and Weibull models dominate the Gaussian, Frechet and Logit Normal as candidates for this distribution. The Weibull reflects a mode and an increasing hazard rate with a fit that is marginally better than the Beta distribution. Figure (2) presents a graph of the histogram of loss rates and the fitted distributions.

We observe from figure (2) the relative closeness of the Weibull and Beta model to each other and the data (displayed as circles). The Gaussian model comes next followed by the Logit normal and the Frechet. This visual ranking of the models is formally confirmed in the χ^2 statistics and corresponding $p - values$. For the latter we used 50 bins with more than 5 observations with the resulting degrees of freedom being 48. The test statistics reported delete the bottom 10% of loss rates and hence we have 38 degrees of freedom.

5 Simulation Results

5.1 Design

A typical simulation run of traces the annual progression of the aggregate premium, loss levels and the fund size at beginning of each year for 10 years for 1000 potential paths. The run produces three 10 by 1000 matrices for the aggregate premium, annual loss level and beginning of year fund size. The annual premiums are defined by equation (1). The aggregate annual loss amount is generated by simulating a Poisson number of failures with mean arrival rate of $\lambda = 20$, for each of which we simulate an asset size from the estimated Frechet distribution, and a loss rate from the estimated Weibull distribution, with the aggregate annual loss being sum over the number of losses of the product of the asset sizes and loss rates. The initial fund size for the next year is defined by equation (2) using the premiums and losses that were generated for the year. On any path for which the funds size reaches the bankruptcy level at the start of some year, the simulation on this path is stopped with the fund size frozen at the bankruptcy level. For the default probability we count the proportion of bankrupt states in the 1000 paths.

For the three components of the loss simulation, the number of failures, the associated asset size and loss rate the details are as follows. For the Poisson number of losses we use the Poisson random number generator from Matlab and generate N_{nm} the number of failures in year n on path m with a constant arrival rate of $\lambda = 20$. This assumption of mean failure rate draws on the failure experience of the FDIC during the post FDICIA period.

For each failure $i \leq N_{nm}$, the asset size we generate a uniform random number $u_{nm}^{(i)}$ for year n on path m and simulate the asset size $A_{nm}^{(i)}$ in accordance with the inverse cumulative distribution method,

$$A_{nm}^{(i)} = c_F \left(-\ln \left(u_{nm}^{(i)} \right) \right)^{-\frac{1}{\alpha_F}} .$$

where, $\alpha_F = 0.94, c_F = 0.0051$.

Similarly, for the loss rate associated with failure i , $l_{nm}^{(i)}$ for year n on path m we generate another independent sequence of uniform random variates $v_{nm}^{(i)}$ with the loss rates now given by the inverse Weibull cumulative distribution function

$$l_{nm}^{(i)} = c_W \left(-\ln\left(1 - v_{nm}^{(i)}\right)\right)^{\frac{1}{\alpha_W}} .$$

where, $\alpha_W = 1.7031, c_W = 0.2404$.

The aggregate annual loss amount for year n on path m, L_{nm} is then

$$L_{nm} = \sum_{i=1}^{N_{nm}} A_{nm}^{(i)} l_{nm}^{(i)}.$$

Table 4 provides the simulated annual loss levels at three quantile points. We observe that the total loss amounts for each path are below the yearly averages of the $1984-1992$ period and above the $1993-2000$ period. Hence, our simulation results depict a world composed of the mixture of these two regimes the FDIC experienced post 1984.

The fund size at the start of year n on path m is C_{nm}. The loss amount for the year is L_{nm}. The premium for the year on this path is

$$P_{nm} = \kappa \left(\max\left(\frac{C_{nm}}{C}, 1\right)\right)^{-\beta} (1 + L_{nm})^{-\gamma}$$

where the policy parameters κ, β, γ, C are prespecified. The fund size at the start of the next year is then

$$C_{n+1,m} = C_{nm} + P_{nm} - L_{nm}.$$

The simulation on a path is stopped the first time C_{n+1}, m is below the bankruptcy threshold of half a billion dollars.

5.2 The Current State

Table 5 reports results for a number of base case alternatives where we assume no rebates and existence of a flat premium structure. Case 1 shows that for the base year of 2000 the fund size was 31 billion dollars with total domestic deposits of 3.3 trillion dollars. Assuming an effective assessment rate of 0.23% generates a flat premium income of 7.65 billion dollars. This effective assessment rate is the rate the FDIC needs to charge per 100 dollar deposits if the mandated reserve to insured deposits is below 1.25%. For a flat premium structure with no countercyclical features β and γ are zero in equation (1). For this setting of the simulation inputs, and given the simulated losses we find that the default probability in 10 years is 6.7%. We should note that currently, because the deposit insurance fund is above the 1.25% threshold, no insurance premium is being collected, which puts the default probability above the 6.7% level.

For a target default probability in 10 years of 5% with no countercyclical features, one may adjust upward the fund size or the aggregate premium level. Case 2 shows that keeping the premium level of 7.65 billion dollars constant it takes a doubling of the fund size to 60 billion dollars to reduce the 10 year default probability below 5%. Alternatively Case 3 demonstrates that keeping the fund size at 31 billion dollars one may raise the level of premiums to 12.5 billion dollars per year (or an effective assessment rate or .375%) to reduce this 10 year default probability below 5%. Finally, Case 4 shows an intermediate possibility where the fund reserve is raised to 40 billion and the effective assessment rate is increased to .3156% to attain the target 5% default probability.

The above simulations employ the same aggregate loss distribution over the ten years as the random number seed is fixed. This distribution is made up of three components cumulated over ten years, and these are the Poisson arrivals, the Frechet assets sizes, and Weibull loss rates each year. We report now on the structure of the resulting aggregate loss distribution. One anticipates that this distribution would be fat tailed given the presence of asset size draws from the Frechet.

For such fat-tailed distributions it is customary to graph them on a log-log plot. Figure (3) presents a plot of the logarithm of the complementary distribution function or the probability of a large loss against the logarithm of the loss level. A linear graph is evidence of a power law and a Pareto tail. We observe from this graph that the relationship is linear and the regression line implies that

$$P(L > x) = \frac{1.2887}{L^{1.2163}} \tag{5}$$

from which we infer a 7.8% probability of a 100 billion dollar loss over the 10 years where L in equation (5) is measured in units of 10 billion dollars. We suspect the large size of these losses is driven to some extent by a uniform arrival rate for large and small bank loss arrival rates and a uniform loss rate distribution independent of asset size. In subsequent revisions we anticipate generalizing the model employed to allow for size dependent arrival rates as well as introducing a negative dependence between between loss rates and asset sizes.

5.3 Countercyclical Trade-offs

We next explore the trade-offs inherent in the design of countercyclical premium systems. We consider a number of sample premium schedules around the base scenario of Case 4 in Table 5 (C_0 = $40 billion fund size and a flat premium level κ =$10.5 billion or an effective assessment rate of .32% that gives a 5% 10 year default probability). The annual loss amounts for the 10 years over the 1000 paths is the same as the one summarized in Table 4.

First, we introduce rebates based on the level of aggregate losses. In other words, in terms of Equation (1), we first assume $\beta = 0$. Suppose we wish to engineer roughly a 50% reduction in the annual $10.5 billion premium level when aggregate losses are $5 billion. The value of γ that satisfies the equation ($50\% = (1 + L)^{-\gamma}$), where $L = .5$, accomplishes this premium reduction.[2] Thus $\gamma = 1.7095$. Such rebate increases the default probability of the fund from 5% to 6.3% at

[2] We express $5 billion as .5 because our calculations are in terms of 10s of billion dollars.

$\kappa = \$10.5$ billion (no rebate zero-loss level annual premium).

We now ask what the effect is on the flat rate premium of introducing such a rebate if we wish to bring back the default probability back to 5%. The flat rate premium associated with a zero level of losses (κ) with loss-rebate structure in place, now rises to $16.5 billion or an effective assessment rate of 0.50%, to maintain the target 10 year default probability at 5%. The actual premium levels however are now indexed to the aggregate loss levels and Table 6 shows premium and fund size for three paths of losses. We observe that premiums are lower when losses are high.

Table 6 also demonstrates how the design can operate. At the beginning of the period the insuring agency announces that if the insurance fund does not experience any losses during the year the aggregate premium charged at the end of the period is $16.5 billion. If aggregate losses during the first year is $2.36 billion as in the $75th$ quartile outcome, then the aggregate premium will be determined as $11.5 = \left(1.65 * (1 + .236)^{-1.7095}\right) * 10$. In other words, $2.36 billion loss level causes the premium level to decrease by $5 billion, which reflects the loss rebate. In terms of fund size, it starts at $40 billion, $11.5 billion premium collected and after $2.36 billion losses $49 billion fund reserve is accumulated.

One interesting observation with Table 6 is that the fund size keeps growing over time to ensure a 5% default probability over ten years. We can adjust the premium structure such that the fund level's growth is slowed down but at the same time the target default probability remains unchanged. One strategy is to collect higher premiums in early years leading to lower levels of fund size in later years. Such design can be accomplished by augmenting the loss rebate with a rebate system associated with the size of the fund. For this purpose we need to determine the parameter β. A policy choice can be to determine the rate at which the excess fund level is to be returned by premium reduction. For example, one choice could be that if the fund size rises to $50 billion, a 25% increase, there will be a 36% rebate in the premiums (the $10 billion excess fund size will be returned to the banking sector roughly in three years). Hence the premium associated with an excess fund size of 25% is $6.72 billion. Such a rebate is organized by $\beta = 2$

$(.672 = 1.05 * (1.25)^{-\beta})$.

With such a rebate system in place, naturally, the annual premium with no loss rebate and for no excess capital state rises. For a 5% default probability target the aggregate premium is $15 billion. Table 7 shows the premium and fund size for a system where it has only rebates for excess fund size but no rebates for losses. To illustrate the operation of the system we again use the 75th quartile results. The insuring agency announces that there will be a rebate of 36% for a 25% fund excess size. At the end of the year 1, fund size equals the beginning size ($40 billion) plus premium collections ($15 billion) less losses for the year ($2.36 billion). Thus end of year fund size is approximately $53 billion. Since fund size is below the benchmark fund size ($C_B = 60$) no rebate will be given and the entire $15 billion is kept. Similarly, there will be no rebate for the second year. However, the third year there is a rebate. The fund size starts with $66 billion. The premium after rebate is approximately $12 billion $\left(15 * (66/60)^{-2}\right)$. As a result ending fund size is approximately $78 billion (beginning fund size of $66 billion plus $12 billion premium minus $0.4 billion losses for the year).

As we can observe the fund size growth is slowed down relative to the results reported in Table 6. Table 8 brings together both loss and excess fund size rebates ($\gamma = 0.5$ and $\beta = 2$). Here, both premium and fund size levels are higher relative to Table 7 but time series pattern follow that of Table 7.

6 Conclusion

This paper provides a mechanism for exploring premium systems that are both responsive to relief in times of crisis and build in a distribution of excess fund sizes while preserving a risk based structure for determining aggregate premiums that ensure viability of the deposit insurance system. For this purpose we study the distributions of assets sizes and loss rates and determine that these are modeled well by the Frechet and Weibull families respectively. Risk Neutralization is seen to naturally occur as a consequence of system viability and results in a growth of the fund size that sets the stage for building in capital rebates that are substantial as we go forward. It remains to explore other criteria for system viability that are more directly connected with law invariant risk measures. Also, we plan to introduce dependence of loss arrival rates and loss rate distribution on asset sizes in the simulation.

7 References

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Table 1: Total assets, loss as a % of total assets, number of bank failures

	Over 5 B	1-5 B	500M-1B	100-500M	50-100M	Under 50M
1984	39,957(7%)	-	513(1%)	1,345(13%)	419(16%)	1,197(23%)
	1	-	1	7	7	64
1985	5,279(7%)			1,075(9%)	454(22%)	1,928(25%)
	1	–	–	5	6	108
1986	–	1,589(14%)	598(23%)	1820(26%)	1468(25%)	2164(27%)
		1	1	10	21	112
1987	–	1,200(0%)	501(13%)	3,284(26%)	1,251(24%)	2,993(27%)
		1	1	15	18	168
1988	18,162(11%)	10,949(13%)	7,717(9%)	10,788(12%)	3,560(15%)	3,280(26%)
	1	4	12	53	51	159
1989	7,181(22%)	6,932(22%)	4,373(16%)	8,739(14%)	1,685(23%)	2,695(25%)
	1	3	6	37	25	135
1990	–	4,144(9%)	1,950(12%)	5,703(24%)	1,488(19%)	2,455(20%)
		2	3	26	22	116
1991	45,591(3%)	9,146(16%)	4,619(22%)	5,943(23%)	1,535(18%)	1,629(21%)
	4	7	6	23	21	66
1992	7,269(10%)	23,704(5%)	3,421(15%)	8,304(10%)	1,456(18%)	1,334(18%)
	1	9	6	32	20	54
1993	–		936(13%)	1,389(20%)	582(21%)	621(20%)
			1	7	8	25
1994	–	–	–	1,217(12%)	77(23%)	111(10%)
				7	1	5
1995	–	–	–	635(10%)	77(13%)	31(29%)
				3	1	2
1996	–	–	–	–	114(19%)	68(25%)
					2	3
1997	–	–	–	–	–	26(19%)
						1
1998	–	–	–	375(60%)	–	53(8%)
				1		2
1999	–	614(0%)	–	115(9%)	157(27%)	61(10%)
		1		1	2	3
2000	–	–	–	114(11%)	239(6%)	38(5%)
				1	3	2

Table 2: Asset Size and Loss Rate Summary Statistics

	Asset Size ($Billions)	Loss Rate (%)
Mean	.751	21.10
Standard Deviation	10.0	12.97
Minimum	.0013	.0053
Maximum	584	93.94
Median	.084	19.63
Lower Quartile	.042	11.64
Upper Quartile	.188	28.80
First Percentile	.008	.3084
Last Percentile	9.67	56.42

Table 3: Results on distributional models for loss rates

Model	Parameter 1	Parameter 2	χ_2	p-value (df=38)
Gaussian	$\mu_G = 0.2066$	$\sigma = 0.1319$	76.06	0.00024
Beta	$\alpha = 1.8454$	$\beta = 6.7546$	49.86	0.0942
Weibull	$\alpha_W = 1.7031$	$C_W = 0.2404$	48.13	0.1256
Frechet	$\alpha_F = 0.9814$	$C_F = 0.109$	570.46	0
Logit Normal	$\mu = -1.5182$	$\Sigma = 0.7175$	122.17	0

Table 4: Simulated annual loss levels at three points of the aggregate loss over ten years

Year	25^{th} Quartile	Median	75^{th} Quartile
1	0.83	0.67	2.36
2	1.08	4.32	1.24
3	0.65	2.72	0.49
4	0.19	2.95	17.95
5	0.14	0.72	1.57
6	1.72	2	1.15
7	1.4	0.57	1.71
8	1.87	2.16	0.33
9	3.46	0.33	0.87
10	1.39	1.45	0.42
Sum	12.75	17.91	28.22

Table 5: Base case alternatives with no rebate system

	Case 1	Case 2	Case 3	Case 4
Fund size ($billion)	31	60	31	40
Domestic deposits ($trillion)	3.3	3.3	3.3	3.3
Effective assessment rate (%)	0.23	0.23	0.38	0.32
Aggregate premium ($billion)	7.65	7.65	12.5	10.5
10-year default probability	6.7	5	5	5

Table 6: Annual premium and fund size with loss rebates only that ensures 5% default probability

Assumed parameters are $C_0 = 40, C_B = 40$, $K = 16.5$, $\gamma = 1.7095$, $\beta = 0$.

	25th Quartile		50th Quartile		75th Quartile	
Year	Premium	Fund Size	Premium	Fund Size	Premium	Fund Size
1	14.4	53	14.8	54	11.5	49
2	13.9	66	8.9	59	13.5	61
3	14.8	80	10.9	67	15.2	76
4	15.9	96	10.6	75	2.8	61
5	16.1	112	14.6	88	12.8	72
6	12.5	123	12.1	99	13.7	85
7	13.2	135	15	113	12.6	96
8	12.3	145	11.8	122	15.6	111
9	9.9	152	15.6	138	14.3	124
10	13.2	164	13.1	150	15.4	139

Table 7: Annual premium and fund size with capital rebates only that ensures 5% default probability

Assumed parameters are $C_0 = 40, C_B = 60,\ K = 15,\ \gamma = 0,\ \beta = 2.$

	25th Quartile		50th Quartile		75th Quartile	
	Premium	Fund Size	Premium	Fund Size	Premium	Fund Size
1	15	54	15	54	15	53
2	15	68	15	65	15	66
3	11.6	79	12.8	75	12.2	78
4	8.6	88	9.6	82	8.8	69
5	7	94	8.1	89	11.3	78
6	6.1	99	6.8	94	8.7	86
7	5.5	103	6.1	99	7.2	92
8	5.1	106	5.5	103	6.4	98
9	4.8	107	5.1	108	5.6	103
10	4.7	111	4.7	111	5.1	107

Table 8: Annual premium and fund size with loss and capital rebates only that ensures 5% default probability

Assumed parameters are $C_0 = 40, C_B = 60,\ K = 19,\ \gamma = 0.5,\ \beta = 2.$

	25th Quartile		50th Quartile		75th Quartile	
	Premium	Fund Size	Premium	Fund Size	Premium	Fund Size
1	18.2	57	18.4	57	17.1	55
2	18	74	15.9	69	18	71
3	11.9	86	12.6	79	13.1	84
4	9.2	95	9.5	86	5.8	72
5	7.6	102	8.9	94	12.3	83
6	6.1	106	7.1	99	9.5	91
7	5.6	111	6.8	105	7.6	97
8	5.1	114	5.6	109	7.2	104
9	4.5	115	5.7	114	6.1	109
10	4.8	118	4.9	118	5.6	114

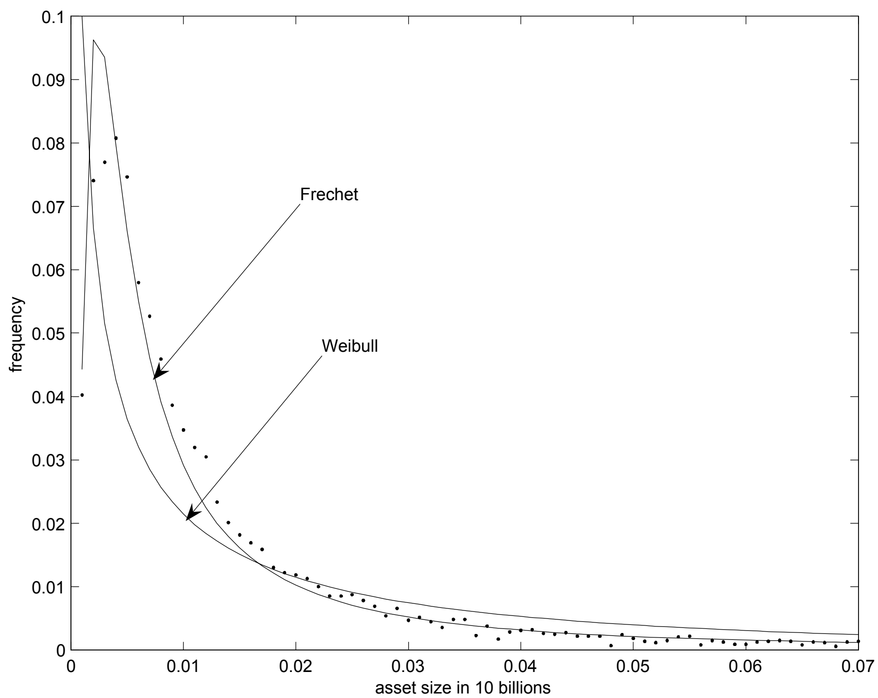

Figure 1: Asset Size Distributions on the Frechet and Weibull Models.

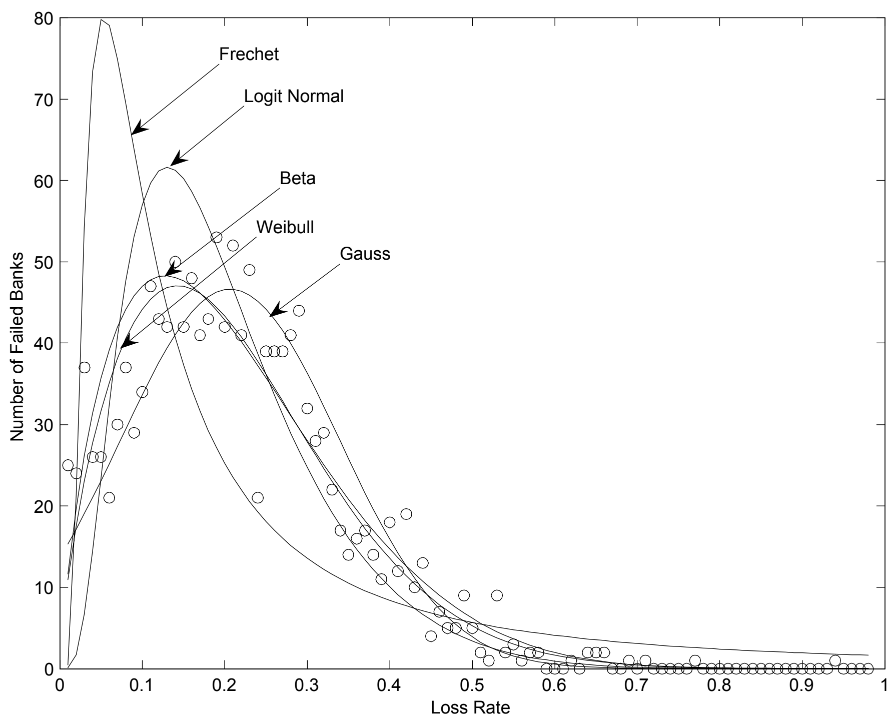

Figure 2: Loss Rate Distributions on the Gaussian, Beta, Weibull, Frechet and Logit Normal Models

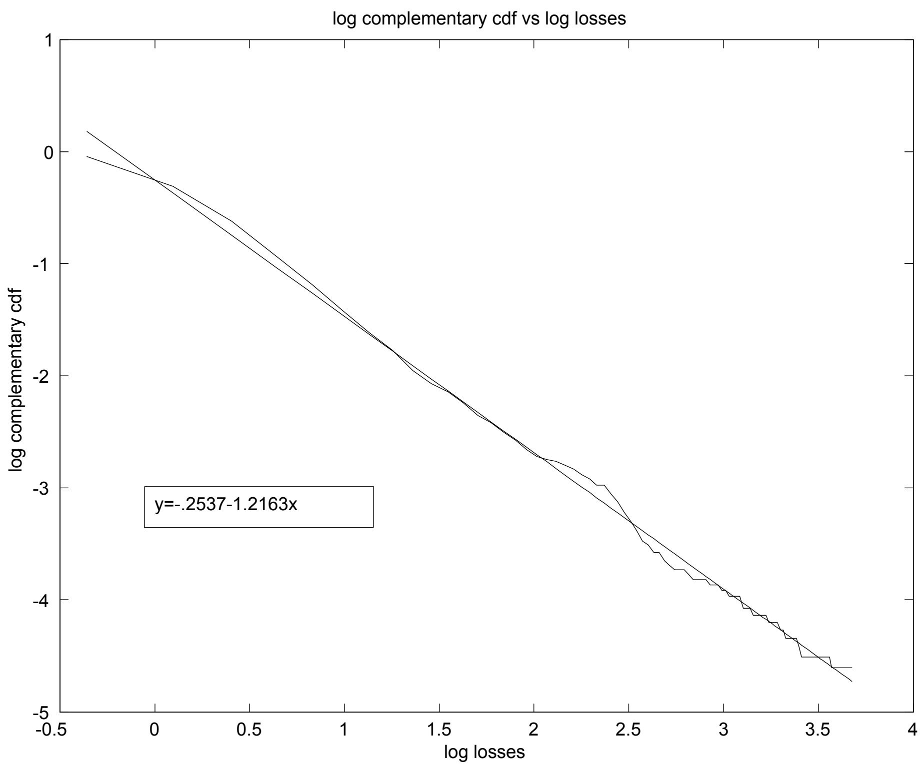

Figure 3: Aggregate Loss Rate Distribution Function. The graph plots logarithm of the complimenatry distribution function against the logarithm of the loss levels. In addition, regression line and the estimate is shown.